STUDENTS' ABILITY IN SOLVING HIGHER ORDER THINKING MATHEMATICS PROBLEMS – A STUDY

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Mathematics is essential for the individual to learn. According to Chen mathematics is very important for various fields with real-life applications, including natural sciences, engineering, medicine, and social sciences. Mathematics can be used to develop skills that involve logical, systematic, critical, careful and creative reasoning skills in communicating ideas or solving problems. Mathematics problems are useful for training students to reflect and analyze mathematics. Therefore, students must be taught how to solve problems with an appropriate problem-solving.

1.2 WHAT IS A PROBLEM?

We distinguish between problems and exercises. An exercise is a question that you know how to resolve immediately. Whether you get it right or not depends on how expertly you apply specific techniques, but you don 't need to puzzle out what techniques to use. In contrast, a problem demands much thought and resourcefulness before the right approach is found.

A good problem is mysterious and interesting. It is mysterious, because at first you don 't know how to solve it. If it is not interesting, you won 't think about it much. If it is interesting, though, you will want to put a lot of time and effort into understanding it.

1.3 WHAT IS A MATHEMATICS PROBLEM?

A mathematics problem is a question or situation that requires mathematical thinking and problem-solving skills to find a solution. These problems can vary widely in complexity, from simple arithmetic calculations to intricate puzzles that involve advanced mathematical concepts. Mathematics problems can be found in various contexts, including textbooks, exams, real-world scenarios, and recreational math puzzles. The process of solving a mathematics problem typically involves understanding the problem, identifying relevant information, choosing appropriate mathematical methods or techniques, performing calculations or manipulations, and interpreting the results to answer the question or address the situation.

Mathematics problems are useful for training students to reflect and analyze mathematics. Therefore, students must be taught how to solve problems with an appropriate problem-solving.

1.4 PROBLEM-SOLVING

Problem solving is an integral part of all mathematics learning. Problem-solving is an essential part of mathematics. Problem-solving is one way to give students the opportunity to develop a deeper understanding of mathematical concepts, English, and various ways to represent mathematical solutions. Students must be encouraged to reflect on their thoughts during the problem-solving process so that they can implement, adjust and modify appropriate strategies to find solutions, so students need opportunities to formulate, solve, and solve complex problems.

Problem solving refers to the process of finding solutions to difficult or complex issues or challenges. It involves identifying a problem, understanding its root causes, brainstorming potential solutions, evaluating those solutions, and implementing the best one. Problem solving is a fundamental skill across various disciplines and is essential for personal, academic, professional, and societal success.

Problem solving is not only about finding solutions but also about developing critical thinking skills, creativity, adaptability, and resilience. It often requires collaboration, communication, and perseverance, especially when dealing with complex or ambiguous problems.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics. Solve problems that arise in mathematics and in other contexts People who see the world mathematically are said to have a "mathematical disposition." Good problem solvers tend naturally to analyze situations carefully in mathematical terms and to pose problems based on situations they see. They first consider simple cases before trying something more complicated, yet they will readily consider a more sophisticated analysis.

A quick answer found by looking at the average time customers had to wait for each company turns out to be misleading. A more careful mathematical analysis involving plotting response times versus time of day reveals a different solution. In this task, a disposition to analyze more deeply leads to a more complete understanding of the situation and a correct solution. Throughout the grades, teachers can help build this disposition by asking questions that help students find the mathematics in their worlds and experiences and by encouraging students to persist with interesting but challenging problems.

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking. By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Posing problems comes naturally to young children: I wonder how long it would take to count to a million? How many soda cans would it take to fill the school building? Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students' problem-solving dispositions by creating and maintaining classroom environments, from prekindergarten on, in which students are encouraged to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop confidence in their abilities and a willingness to engage in and explore problems, and they will be more likely to pose problems and to persist with challenging problems.

1.4.1 BUILD NEW MATHEMATICAL KNOWLEDGE THROUGH PROBLEM SOLVING

How can problem solving help students learn mathematics? Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning. With young children, most mathematical concepts can be introduced through problems that come from their worlds. For example, suppose second graders wanted to find out whether there are more boys or girls in the four second-grade classes. To solve this problem, they would need to learn how to gather information, record data, and accurately add several numbers at a time. In the middle grades, the concept of proportion might be introduced through an investigation in which students are given recipes for punch that call for different amounts of water and juice and are asked to determine which is "fruitier." Since no two recipes yield the same amount of juice, this problem is difficult for students who do not have an understanding of proportion. As various ideas are tried, with good questioning and guidance by a teacher, students eventually converge on using proportions. In high school, many areas of the curriculum can be introduced through problems from mathematical or applications contexts.

1.5 STEPS IN EFFECTIVE PROBLEM SOLVING

Effective problem solving typically involves several key steps:

I. **Identifying the problem:** Clearly defining and understanding the issue at hand is crucial. This might involve gathering information, asking questions, and analyzing the situation.

II. **Analyzing the problem:** Once the problem is identified, it's important to break it down into smaller, more manageable components. This can help in understanding the underlying causes and identifying potential solutions.

III. **Generating possible solutions:** Brainstorming and exploring various options for solving the problem. Creativity and thinking outside the box can be valuable during this stage.

IV. **Evaluating solutions:** Assessing the potential effectiveness of each solution based on various criteria such as feasibility, cost, time required, and potential outcomes. It's important to consider both short-term and long-term implications.

V. **Selecting the best solution:** Choosing the most suitable solution based on the evaluation process. This might involve a compromise or combination of different solutions.

VI. **Implementing the solution:** Taking action to put the chosen solution into practice. This may involve planning, organizing resources, and executing the plan effectively.

VII. **Evaluating the outcome:** Reflecting on the results of the implemented solution to determine its success and identify any lessons learned. This feedback loop can inform future problem-solving efforts.

1.6 STAGES OF PROBLEM SOLVING KRULIK AND RUDNICK

Based on **Krulik and Rudnick** there are five stages of problem solving, namely:

1. **Read and think**, at this stage the problem is analyzed, the question is identified, the relationship of each information in the problem is determined, and the problem is changed into the language that is easy to understand;

2. **Explore and plan**, at this stage, the data is analyzed whether there is enough information that can be used for problem-solving, then the data can be arranged in tables, images, graphs, models, etc. So that a plan was developed to get the answer;
3. **Select a strategy**, at this stage a strategy is estimated that can be used, such as finding a pattern, working backward, guessing and testing and simulating or experimenting so that it can provide direction for solving problems that must be done in finding the answer;
4. **Find an answer:** this stage includes solution estimation, use of computational capabilities, and the use of algebra and geometry skills;
5. **Reflect and extend**: the solution that has been obtained from the previous stage is reexamined, then determines the alternative solution and makes an expansion or general.

Problem-solving occurs when students can think toward the expected solution. Thinking ability has a role in interpreting the situation in the problem-solving process so that when students face unusual problems, it takes high-level thinking skills.

1.7 APPLY AND ADAPT A VARIETY OF APPROPRIATE STRATEGIES TO SOLVE PROBLEMS

Of the many descriptions of problem-solving strategies, some of the best known can be found in the work of Pólya (1957). Frequently cited strategies include using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backward, guessing and checking, creating an equivalent problem, and creating a simpler problem. An obvious question is, how should these strategies be taught? Should they receive explicit attention, and how should they be integrated with the mathematics curriculum? As with any other component of the mathematical tool kit, strategies must receive instructional attention if students are expected to learn them. In the lower grades, teachers can help children express, categorize, and compare their strategies. Opportunities to use strategies must be embedded naturally in the curriculum across the content areas. By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them. By high school, students should have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies.

Young children's earliest experiences with mathematics come through solving problems. Different strategies are necessary as students experience a wider variety of problems. Students must become aware of these strategies as the need for them arises, and as they are modeled during classroom activities, the teacher should encourage students to take note of them. For example, after a student has shared a solution and how it was obtained, the teacher may identify the strategy by saying, "It sounds like you made an organized list to find the solution. Did anyone solve the problem a different way?" This verbalization helps develop common language and representations and helps other students understand what the first student was doing. Such discussion also suggests that no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations.

1.8 MONITOR AND REFLECT ON THE PROCESS OF MATHEMATICAL PROBLEM SOLVING

Effective problem solvers constantly monitor and adjust what they are doing. They make sure they understand the problem. If a problem is written down, they read it carefully; if it is told to them orally, they ask questions until they understand it. Effective problem solvers plan frequently. They periodically take stock of their progress to see whether they seem to be on the right track. If they decide they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach. Research (Garofalo and Lester 1985; Schoenfeld 1987) indicates that students' problem-solving failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know.

Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al. 1999). Such reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as "Before we go on, are we sure we understand this?" "What are our options?" "Do we have a plan?" "Are we making progress or should we reconsider what we are doing?" "Why do we think this is true?" Such questions help students get in the habit of checking their understanding as they go along. This habit should begin in the lowest grades. As teachers maintain an environment in which the development of understanding is consistently monitored through reflection, students are more likely to learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems.

1.9 MEANINGFUL LEARNING

Meaningful learning provides students with the knowledge and cognitive processes they need for successful problem solving. Problem solving occurs when a student devises a way of achieving a goal that he or she has never previously achieved, that is, of figuring out how to change a situation from its given state into a goal state (Duncker, 1945; Mayer, 1992). Two major components in problem solving are problem representation-in which a student builds a mental representation of the problem-and problem solution-in which a student devises and carries out a plan for solving the problem (Mayer, 1992).

1.10 TAXONOMY

A taxonomy is a special kind of framework. In a taxonomy the categories lie along a continuum. The continuum (e.g., the wave frequencies underlying color, the atomic structure underlying the periodic table of the elements) becomes one of the major organizing principles of the framework. In our Taxonomy we are classifying objectives. A statement of an objective contains a verb and a noun. The verb generally describes the intended cognitive process. the noun generally describes the knowledge students are expected to acquire or construction Consider the following example: "The student will learn to distinguish (the cognitive process) among confederal, federal, and unitary systems of government (the knowledge).

1.11 CATEGORIES OF THE COGNITIVE PROCESS DIMENSION IN B. S. BLOOM TAXONOMY

In life, objectives help us to focus our attention and our efforts; they indicate what we want to accomplish. In education, objectives indicate what we want students to learn; they are "explicit formulations of the ways in which students are expected to be changed by the educative process". Objectives are especially important in teaching because teaching is an intentional and reasoned act. Teaching is intentional because we always teach for some purpose, primarily to facilitate student learning. Teaching is reasoned because what teachers teach their students is judged by them to be worthwhile.

The reasoned aspect of teaching relates to what objectives teachers select for their students. The intentional aspect of teaching concerns how teachers help

students achieve the teachers' objectives, that is, the learning environments the teachers create and the activities and experiences they provide. The learning environments, activities, and experiences should be aligned with, or be consistent with, the selected objectives.

Bloom's taxonomy is a hierarchical model used to classify educational learning objectives into levels of complexity and specificity. Originally proposed by educational psychologist Benjamin Bloom in 1956, the taxonomy has been revised over the years. The categories of the cognitive process dimension are intended to provide a comprehensive set classification for those student cognitive processes that are included in objectives. The categories range from the cognitive processes most commonly found in objectives, those associated with Remember, through

Understand and Apply, to those less frequently found, Analyze, Evaluate, and Create.

- **Remember** means for retrieve relevant knowledge from long-term memory. Understand is defined as constructing the meaning of instructional messages, including oral, written, and graphic communication.
- **Apply** means carrying out or using a procedure in a given situation.
- **Analyze** is breaking material into its constituent parts and determining how the parts are related to one another as well as to an overall structure or purpose.
- **Evaluate** means making judgments based on criteria and or standards.
- **Create** is putting elements together to form a novel, coherent whole or to make an original product.

Each of the six major categories is associated with two or more specific cognitive processes, in all, also described by verb forms. To differentiate the specific cognitive processes from the six categories, the specific cognitive processes take the form of gerunds, ending in "ing." Thus, recognizing and recalling are associated with Remember; interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining are associated with Understand; executing and implementing with Apply; and so on.

1.11.1 REMEMBER

Remembering involves retrieving relevant knowledge from long term memory. The two associated cognitive processes are recognizing and recalling. The relevant knowledge may be Factual, Conceptual, Procedural, or Metacognitive, or some combination of these.

To assess student learning in the simplest process category, the student is given a recognition or recall task under conditions very similar to those in which he or she learned the material. Little, if any, extension beyond those conditions is expected.

Remembering knowledge is essential for meaningful learning and problem solving as that knowledge is used in more complex tasks. For example, knowledge of the correct spelling of common English words appropriate to a given grade level is necessary if the student is to master writing an essay. Where teachers Concentrate solely on rote learning, teaching and assessing focus solely on remembering elements or fragments of knowledge, often in isolation from their context.

1.11.2 UNDERSTAND

As we indicated, when the primary goal of instruction is to promote retention, the focus is on objectives that emphasize Remember. When the goal of instruction is to promote transfer, however, the focus shifts to the other five cognitive processes, understand through Create. Of these, arguably Ihe largest category of transfer-based educational objectives emphasized in schools and colleges is Understand. Students are said to Understand when they are able to construct meaning from instructional messages, including oral, written, and graphic communications, however they are presented to students: during lectures, in books, or on computer monitors. Examples of potential instructional messages include an in-class physics demonstration, a geological formation seen on a field trip, a computer simulation of a trip through an art museum, and a musical work played by an orchestra, as well as numerous verbal, pictorial, and symbolic representations on paper.

Students understand when they build connections between the "new" knowledge lo be gained and their prior knowledge. More specifically, the incoming knowledge is integrated with existing schemas and cognitive frameworks. Since concepts are Ihe building blocks for These schemas and frameworks, Conceptual knowledge provides a basis for understanding. Cognitive processes in the category of Understand include interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.

1.11.3 APPLY

Apply involves using procedures to perform exercises or solve problems. Thus, apply is closely linked with Procedural/ knowledge. An exercise is a task for which the student already knows the proper procedure to use, so the student has developed a fairly routinized approach to it. A problem is a task for which the student initially does not know what procedure to use, so the student must locate a procedure to solve the problem. The Apply category consists of two cognitive processes: executing-when the task is an exercise (familiar)-and implementing when the task is a problem (unfamiliar).

When the task is a familiar exercise, students generally know what Procedural knowledge to use. When given an exercise (or set of exercises), students typically perform the procedure with little thought. For example, an algebra student confronted with the 50th exercise involving quadratic equations might simply "plug in the numbers and turn the crank."

When the task is an unfamiliar problem, however, students must determine what knowledge they will use. If the task appears to call for Procedural knowledge and no available procedure fits the problem situation exactly, then modifications in selected Procedural knowledge may be necessary. In contrast to executing, then, implementing requires some degree of understanding of the problem as well as of the solution procedure. In the case of implementing, then, to understand conceptual knowledge is a prerequisite lo being able to app/y procedural knowledge.

1.11.4 ANALYZE

Analyze involves breaking material into its constituent parts and determining how the parts are related to one another and to an overall structure. This process category includes the cognitive processes of differentiating, organizing, and attributing.

Objectives classified as Analyze include learning to determine the relevant or important pieces of a message (differentiating), the ways in which the pieces of a message are organized (organizing), and the underlying purpose of the message (attributing). Although learning to Analyze may be viewed as an end in itself, it is probably more defensible educationally to consider analysis as an extension of Understanding or as a prelude to Evaluating or Creating.

Improving students' skills in analyzing educational communications is a goal in many fields of study. Teachers of science, social studies, Ihe humanities, and the arts frequently give "learning to analyze" as one of their important objectives. They may, for example, wish to develop in their students the ability to:

- distinguish fact from opinion (or reality from fantasy);
- connect conclusions with supporting statements;
- distinguish relevant from extraneous material;
- determine how ideas are related to one another;
- ascertain the unstated assumptions involved in what is said;
- distinguish dominant from subordinate ideas or themes in poetry or music and
- find evidence in support of the author's purposes.

1.11.5 EVALUATE

Evaluate is defined as making judgments based on criteria and standards. The criteria most often used are quality, effectiveness, efficiency, and consistency.

They may be determined by the student or by others. The standards may be either quantitative (i.e., ls this a sufficient amount?) or qualitative (i.e., ls this good enough?). The standards are applied to the criteria (e.g., ls this process sufficiently effective? ls this product of sufficient quality?). The category Evaluate includes the cognitive processes of checking judgements about the internal consistency) and critiquing judgements based on external criteria).

It must be emphasized that not all judgments are evaluative. For example, students make judgments about whether a specific example fits within a category. They make judgments about the appropriateness of a particular procedure for a specified problem. They make judgments about whether two objects are similar or different. Most of the cognitive processes, in fact, require some form of judgment. What most clearly differentiates Evaluate as defined here from other judgments made by students is the use of standards of performance with clearly defined criteria. ls this machine working as efficiently as it should be? Is this method the best way to achieve the goal? Is this approach more cost effective than other approaches? Such questions are addressed by people engaged in Evaluating.

1.11.6 CREATE

Create involves putting elements together to form a coherent or functional whole. Objectives classified as Create have students make a new product by mentally reorganizing some elements or parts into a pattern or structure not clearly present before. The processes involved in Create are generally coordination with the student’s previous learning experiences. Although create requires creative thinking on the part of the student, this is not completely free creative expression unconstrained by the demands of the learning task or situation.

To some persons, creativity is the production of unusual products, often as a result of some special skill. Create, as used here, however, although it includes objectives that call for unique production, also refers to objectives calling for production that all students can and will do. If nothing else, in meeting these objectives, many students will create in the sense of producing their own synthesis of information or materials to form a new whole, as in writing, painting, sculpting, building, and so on.

Although many objectives in the Create category emphasize originality, educators must define what is original or unique. Can the term unique be used to describe the work of an individual student (e.g., "This is unique for Adam Jones") or is it reserved for use with a group of students (e.g., "This is unique for a fifth-grader")? It is important to note, however, that many objectives in the create category do not rely on originality or uniqueness. The teachers' intent with these objectives is that students should be able to synthesize material into a whole. This synthesis is often required in papers in which the student is expected to assemble previously taught material into an organized presentation.

1.12 What Is Higher-Order Thinking?

we consider the kinds of higher-order thinking that are stated or implied in state content standards and classroom learning objectives. Definitions that I find helpful fall into three categories:

1) Transfer of learning

2) Critical thinking

3) Problem solving

Transfer of learning:

Two of the most important educational goals are to promote retention and to promote transfer (which, when it occurs, indicates meaningful learning) . . . retention requires that students remember what they have learned, whereas transfer requires students not only to remember but also to make sense of and be able to use what they have learned.

Critical thinking:

Critical thinking is reasonable, reflective thinking that is focused on deciding what to believe or do. Critical thinking as "artful thinking", which includes reasoning, questioning and investigating, observing and describing, comparing and connecting, finding complexity, and exploring viewpoints.

Problem solving:

A student incurs a problem when the student wants to reach a specific outcome or goal but does not automatically recognize the proper path or solution to use to reach it. The problem to solve is how to reach the desired goal. Because a student cannot automatically recognize the proper way to reach the desired goal, she must use one or more higher-order thinking processes. These thinking processes are called problem solving. (Nitko & Brookhart, 2007, p. 215)

As any taxonomy of higher-order thinking skills shows, pulling a concept apart and discussing its various aspects is one way of understanding it. Think of this book as an analysis of classroom assessment of higher-order thinking.

1.13 HIGH ORDER THINKING SKILLS BASED ON BLOOM'S TAXONOMY OF EDUCATIONAL OBJECTIVES

High Order Thinking Skills (HOTS) is a concept of education reform based on **Bloom's Taxonomy** with the idea that some types of learning require more cognitive processing than others, but also have more general benefits and involve complex judging skills such as problem-solving. Let us discuss categories of the cognitive process dimensions in details.

A key feature of effective learning is the development of higher order thinking. indeed, it finds voice in Based on Bloom's Taxonomy of Educational Objectives, Higher order thinking is not a new concept and it concerns synthesis, evaluation, interpretation, hypothesizing, prediction, conjecture, critical thinking and judgement. It is complex, and involves reflection, self-regulation, testing of ideas, and problem solving.

The highest form of 'cognitive engagement' is where learners plan and manage their own learning and exercise considerable autonomy, together with reflection on the learning experience and the incorporation of new knowledge into existing knowledge. Hence planning, living with uncertainty, prediction, making meaning, adopting multiple perspectives on an issue are all characteristics of higher order thinking. Critical thinking involves finding information suitable for a specified purpose, analysing and evaluating arguments, information and sources, separating fact from opinion, exposing unstated assumptions, weighing evidence, evaluating the logic of the argument and the conclusions.

Lower-order thinking skills (LOTS) and higher-order thinking skills (HOTS) are two categories used to describe different levels of cognitive processing and complexity in learning and problem-solving. These categories are often associated with Bloom's taxonomy, which delineates cognitive skills from simple recall to complex analysis and creation.

LOTS are basic cognitive skills that involve the recall or reproduction of information without significant processing or analysis. They typically include tasks such as memorization, repetition, and recognition of information. Examples of LOTS include remembering facts, defining terms, identifying elements, and performing routine procedures. LOTS are essential for laying foundations and acquiring basic knowledge in a subject area.

HOTS involve more advanced cognitive processes that require critical thinking, analysis, synthesis, and evaluation of information. They go beyond mere recall and involve deeper understanding, application, and creation of knowledge. Examples of HOTS include analyzing data, solving complex problems, making connections between ideas, evaluating arguments, and generating new ideas. HOTS are essential for developing critical thinking skills, problem-solving abilities, and creativity.

In education, fostering both LOTS and HOTS is important for comprehensive learning and skill development. While LOTS provide the foundational knowledge and basic understanding necessary for higher-level thinking, HOTS enable students to apply, analyze, and evaluate information in meaningful ways, leading to deeper learning and greater intellectual engagement. Effective teaching strategies often aim to scaffold students' learning experiences to gradually build from lower-order to higher-order thinking skills. The dimensions of HOTS based on Bloom's Taxonomy revised by Anderson and Krathwohl namely, analyze, evaluate, and create.

Higher order thinking is appropriate for all ages, abilities and levels of student; it is not something that is addressed after the lower order skills have been learned, but is simultaneous with them. Learning lower order skills and knowledge is achieved most effectively when it is in the context of learning and using higher order skills. Effective teachers use powerful teaching strategies, e.g. those which are suitably flexible to be tailored to the needs of learners, which encourage student talk and dialogue, and which encourage divergent thinking in which there is no single right answer or solution, i.e. higher order thinking.

1.14 SIGNIFICANCE OF LEARNING ACHIEVEMENT

Learning Achievement or Academic Achievement at higher secondary school is of crucial importance,

a) To develop student's self confidence

b) To test student's cognitive skill

c) To develop student's self esteem

d) To develop student's self-regulation

e) To decide student's career

1.15 CONCLUSION

In this chapter we discussed about problem, problem solving ability, stages for problem solving, blooms taxonomy, higher order thinking skill and learning achievement. It helps to understand that learning achievement is associated with problem solving ability. In this chapter we explored the deeper understanding of hot questions and how it influences by problem solving ability.

CHAPTER 2 REVIEW OF LITERATURE

STUDENTS' ABILITY IN SOLVING HIGHER ORDER THINKING MATHEMATICS PROBLEM – A STUDY

2.1 INTRODUCTION

Effective problem solvers constantly monitor and adjust what they are doing. They make sure they understand the problem. If a problem is written down, they read it carefully; if it is told to them orally, they ask questions until they understand it. Effective problem solvers plan frequently. They periodically take stock of their progress to see whether they seem to be on the right track. If they decide they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach.

Heffington, Deon Victoria (2023) studied that educational systems worldwide underscore the importance of developing higher-order thinking skills (HOTS) to prepare students for the new challenges of the XXI century. Some pressing issues faced by educators include the ambiguity of the construct; the implementation of HOTS in classroom practices; and the implications for teaching students from linguistically and culturally diverse backgrounds. Framed in Culturally and Linguistically Responsive Teaching, this article reports on a qualitative case study that explored how two elementary school teachers developed HOTS with emerging multilingual students. Using the constant comparative method (CCM), data collection included observations and artifacts; data analysis included open, axial coding, and category formation. Findings show that: (a) HOTS were framed as subskills in instructional practices; (b) teachers used multiple activities to develop HOTS, progressing from lower to higher-order thinking; and (c) teachers differentiated instruction considering students' language level and background experiences, but did not account for the additional layers of complexity when interrelating HOTS to language. This article builds on existing theories and frameworks of HOTS, provides examples of activities for developing HOTS, and offers recommendations for teaching HOTS to multilingual learners.

Rayner Bin Tangkui; Tan Choon Keong (2023) investigated the effect of using Minecraft on Year 5 pupils' higher-order thinking skills (HOTS) in fractional problems-solving. A quasi-experimental pretest and posttest non-equivalent groups design was used. The study sample involved 65 Year 5 pupils from two intact classes which consists of 31 pupils as the treatment group and the other 34 pupils as the control group. Minecraft was used as the intervention in the teaching and learning of fractions in the treatment group. The research data was collected through the administration of pretest and post-test while the data was analysed using paired sample t-test and independent sample t-test. The research resulted in several findings. Among them is the significant difference in the ability to solve fractional problems which requires the use of HOTS between pupils who were exposed to the teaching and learning of fractions using Minecraft and pupils who were exposed to the teaching and learning of fractions using conventional methods. This study has proven that the use of Minecraft in the teaching and learning of fractions has the potential to facilitate and enhance pupils' level of HOTS.

Bai, Yongxiao; Liang, Haili; Qi, Chunxia; Zuo, Siyu (2023) examined that based on Bloom's Taxonomy of Educational Objectives and the assessment framework of PISA, this study aimed to develop a three-dimensional assessment framework to measure junior high school students' higher order thinking skills in mathematics in China. A total of 28,153 eighth graders from 11 districts and counties in Z city were involved in the test. The results showed that (1) in the "Mathematical ability" dimension, students scored the lowest in "Problem-solving"; (2) in the "Cognitive level" dimension, students scored the lowest in "Evaluate"; (3) there was little difference between students' scores on problems in "Realistic" and "Intramathematical situations"; (4) there were significant differences in the higher order thinking skills of students at different academic levels; and (5) there were no gender differences in students' higher order mathematical thinking skills (HOMTS). Implications for assessment in HOMTS and suggestions for future study are discussed.

Heron, Marion; Palfreyman, David M. (2023) studied a key purpose of higher education seminars is to support higher-order thinking, yet empirical evidence of how this is evidenced and scaffolded in higher education remains scarce. Building on previous work on identifying rhetorical and linguistic devices for argumentation, we found that higher-order thinking can be evidenced through using metaphors, linking ideas to personal experiences and emotional connections. Findings also suggest that the types of tutor questioning can support (or not) how students evidence their claims and demonstrate higher-order thinking. We conclude with recommendations for practice including greater teacher and student metacognitive awareness of the features of quality seminar discourse.

Aydin, Utkun; Birgili, Bengi (2023) pointout that Internationally, mathematics education reform has been directed toward characterizing educational goals that go beyond topic/content/skill descriptions and develop students' problem solving. The Revised Bloom's Taxonomy and MATH (Mathematical Assessment Task Hierarchy) Taxonomy characterize such goals. University entrance examinations have been seen as one way of accomplishing these goals and influence learning, teaching, and assessment in mathematics. The present study analyzed mathematics items (N = 1077) in Turkish university entrance examinations in 1998-2013 and objectives (N = 621) in mathematics curricula in 2005, 2011, and 2013 to determine the extent to which they represent the dimensions/categories of these taxonomies and the degree to which items are aligned with objectives in terms of reflecting the dimensions/categories of these taxonomies. The findings reveal that the items demand, to a large extent, automated computational skills; this is also evident in the relevant mathematics curricula. Implications for practice are discussed and could play a role in reforming assessment.

Yigletu, Ayanaw; Michael, Kassa; Atnafu, Mulugeta (2023) explained that teacher education, enhancing pre-service teachers' higher-order thinking skills (HOTs) is very crucial. The effect of comprehensive professional development in assessment for learning (AfL) on pre-service teachers' HOTs in an algebra course was investigated using a nonequivalent group quasi-experimental design. A total of 129 pre-service teachers who took the Fundamental Concepts of Algebra course from three different teacher education colleges [TECs] selected randomly from ten Ethiopian TECs, participated in the study. Among these students who attended the course, a group of 52 pre-service teachers with three mathematics teacher educators who gave them the course engaged in comprehensive training and professional development on AfL, while the remaining pre-service teachers were attending the course the usual way in two different TEC. A one-way ANOVA was used to determine whether there was a mean difference among the three groups and three achiever levels on their HOTs scores. The result showed that there is a statistically significant mean difference in the HOTs scores between pre-service teachers in the treatment group and the comparison groups. The results also showed that there was a statistically significant mean difference among the three achiever levels (low, medium, and high) in their HOTs scores in the treatment group before the intervention, but there was no statistically significant mean difference after the intervention. The implications of the results and recommendations are discussed.

Jahudin, Janet; Siew, Nyet Moi (2023) discussed that diagnostic tests have been developed previously to measure algebraic thinking skills; however, the tests do not specifically address algebraic problem-solving. Thus, an Algebraic Thinking Test (ATT) Instrument was developed to measure algebraic thinking skills in problem-solving involving linear equations. ATT comprises nine open-ended questions with three algebraic thinking constructs: Generalized Arithmetic, Functions and Modelling. Generalized arithmetic involves students in efficient calculation and generalization; functional involves identifying number patterns, while modelling involves solving open-ended problems, identifying similarities, and performing calculations involving variables. This study is meant to determine the quality of ATT instruments through the validity and reliability analysis using the Rasch Measurement Model. The sample consisted of 120 seventh graders aged 12 to 13, selected from two secondary schools in the Tuaran district, Malaysia. The instrument was found to have a strong dimensionality and high construct validity. The reliability of Cronbach Alpha (KR-20) demonstrated a value of 0.90 (very high), and item and respondent reliability of 0.98 (excellent) and .86 (good), respectively, with an item separation index of 6.29 and 2.45 for the person separation index. ATT has good validity and high reliability in measuring algebraic thinking skills among seventh graders in secondary schools.

Ayanaw Yigletu (2023) explained that the effect of comprehensive professional development in assessment for learning (AfL) on pre-service teachers' HOTs in an algebra course was investigated using a nonequivalent group quasi-experimental design. A total of 129 pre-service teachers who took the Fundamental Concepts of Algebra course from three different teacher education colleges [TECs] selected randomly from ten Ethiopian TECs, participated in the study. Among these students who attended the course, a group of 52 pre-service teachers with three mathematics teacher educators who gave them the course engaged in comprehensive training and professional development on AfL, while the remaining pre-service teachers were attending the course the usual way in two different TEC. A one-way ANOVA was used to determine whether there was a mean difference among the three groups and three achiever levels on their HOTs scores. The result showed that there is a statistically significant mean difference in the HOTs scores between pre-service teachers in the treatment group and the comparison groups. The results also showed that there was a statistically significant mean difference among the three achiever levels (low, medium, and high) in their HOTs scores in the treatment group before the intervention, but there was no statistically significant mean difference after the intervention. The implications of the results and recommendations are discussed.

Charanjit Kaur S. Singh (2023) Cultivation of students' higher-order thinking ability has become the main agenda of the education curriculum. The transfer of knowledge pertaining to higher-order thinking by teachers to the students can prepare the latter with the necessary attributes for the 21st century. The present study is aimed at exploring Malaysian secondary ESL (English as a Second Language) school teachers' self-assessment of, and perceptions on the higher-order thinking skills practices for teaching writing. Using a mixed-method research design, the validated 30-item five-point Likert scale questionnaire with an open-ended question was administered to a group of respondents consisting of 72 ESL teachers. The findings indicated that the overall mean score of ESL teachers' self-assessment of using higher-order thinking skills practices for teaching writing was at a high level. Meanwhile, their perceptions on the integration of higher-order thinking skills in the teaching of writing include the concern of students' low proficiency, difficulties in implementing HOTS in writing, poor participation by passive students, and teachers' attitude towards using HOTs for teaching writing. This study suggests that fostering and creating awareness of mastering the elements of HOTS can benefit both the teachers and the students. Teachers have to be creative and innovative in their teaching so that the students can be given the opportunity to showcase their knowledge, skills and abilities in the quest to imbue them with the 21st century life skills.

Liu, Dongping and Zhang, Hai (2022) compared traditional teacher-centered instruction and a WeChat based flipped classroom to investigate student achievements and higher order thinking skills, such as critical thinking and problem-solving. The WeChat based flipped classroom adopted a Structured Query Language (SQL) lesson from an information technology course. The 94 university students were divided into an experimental group (n = 50) and a control group (n = 44), and an achievement test, the critical thinking disposition inventory--Chinese version (CTDI-CV), and a questionnaire and problem-solving survey used in both groups. It was found that the flipped classroom student achievements and higher order thinking skills were significantly better than their peers who had been taught in a traditional classroom environment. This study provides empirical evidence that higher education achievement and higher order thinking skills can be improved using a WeChat-based flipped classroom approach and is a valuable reference for both instructors and educators on the use of social software.

Hariadi, Bambang; Jatmiko, Budi and others (2022) analysed that the effectiveness of the blended web mobile learning model in improving Higher Order Thinking Skills-based learning outcomes of high school students. This research is a quasi-experiment design. The research sample was 137 students from 2 senior high schools in East Java Province - Indonesia. Before implementing the BWML model using MoLearn, both groups of students were tested (pre-test) and after the learning was complete, students were again given the same test (post-test). The collected data were analyzed using the Wilcoxon test; calculation of n-gain; and the Mann-Whitney U test. The results shows that biology learning using the BWML Model is effective in improving student learning outcomes as indicated by: (1) an increase in student learning outcomes which is statistically significant at [alpha]=5%, (2) the average of n-gain is in the medium category, and (3) the mean of n-gain has no difference in the two groups of students.

Utomo, S. W.; Joyoatmojo and others (2022) obtained a learning model that can improve higher order thinking skills in financial reporting of accounting. The model was developed using four stages of research and development (R&D): 1) preliminary research; 2) development; 3) testing for limited scale; 4) testing for wide-scale. The population of this research is the Accounting Vocational School in Madiun. For limited scale test, it was concluded that there were significant differences in HOTS and learning achievement scores before and after implementation of the model (sig. < 0.05), but Ngain for HOTS and learning achievement 0.110 (low) and 0.08 (stable). For test wide-scale class, it was concluded that there were significant differences in HOTS and learning achievement in experiment class (sig. < 0.05), with Ngain for HOTS and learning achievement 0.183 (low) and 0.30 (medium) for SMKN 2 Jiwan and 0.11 (low) and 0.105 (low) for SMK 5 Madiun.

Azid, Nurulwahida; Ali, Ruzlan Md. and others (2022) investigated the effect of incorporating higher order thinking skills (HOTS) in a school-based assessment (SBA) on mathematics achievement among students. A mixed-method case study design was used to assess a sample size of 158 students and three mathematics teachers. The students' proficiency in mathematics was determined by using the standard based curriculum for secondary schools ("kurikulum standard sekolah menengah/KSSM").

Mathematics assessment paper that was assiduously organized by incorporating HOTS questions. The students' thoughts on the incorporation of HOTS in the SBA were obtained through one-to-one face-to-face semi-structured interviews. The quantitative findings, which were analyzed using descriptive and regression tests, showed that 11.9% of the achieved mathematics score was contributed by incorporating HOTS in SBA, while 7.7% was contributed by SBA mastery level in mathematics. Students' maturity and misconceptions about math are among the challenges in implementing HOTS in SBA. The challenges in applying HOTS in SBA had a positive effect on teachers' pedagogical approach in a sense that they could devise a new strategy for meeting the needs of students and teach topics in alternative ways.

Kevin Michael Jennings (2021) found the lived experiences of Serious Game designers. More specifically, how they incorporate higher-order thinking skills, as defined by Revised Bloom's Taxonomy (2001), and assessing these skills. The main research questions are: "Which Serious Game design principles allow for the incorporation of higher-order thinking skills?" and "How are Serious Game designers assessing higher-order thinking skills in their games?" This qualitative phenomenological study was based on interviews with Serious Game designers. The sampling of this study was criterion sampling, in which participants are chosen based on the fact that they meet all the necessary criteria. The interview setting was over Zoom with Serious Game professionals from North America and Europe. Interviews were recorded with QuickTime Audio Recorder, transcribed with the website TranscribeMe, and analyzed on Microsoft Excel. The study's participants gave various answers to the three sub-questions but showed several areas of overlap or saturation. Regarding sub-question A, principles of Serious Game design include incorporating learning objectives and understanding the context of the audience. Regarding sub-questions B, to incorporate higher-order thinking skills into a Serious Game, the designer must create a sufficient system and give agency to the players. Lastly, sub-question C, regarding the assessment of higher-order thinking skills, are primarily assessed through qualitative means. [The dissertation citations contained here are published with the permission of ProQuest LLC. Further reproduction is prohibited without permission.

Lee, Mi-Suk; Chae, Soo Eun (2021) mentioned that scholars have defined higher-order thinking (HOT) skills as a cognitive ability to interpret, analyze, arrange, and extend newly obtained and learned information (Lewis & Smith, 1993; Newmann, 1991a). Higher-order thinking skills draw much public attention, because they have been deemed to be an essential determinant of contemporary educational outcomes. Due to a flood of information in this era due to easy and fast internet access and synchronous social networking, skills such as how to select, organize, and critically analyze sources are more valued than skills to acquire knowledge and information (Alexander et al., 2011). The aforementioned informational flood is even more emphasized in South Korea, where fast internet connections and mobile technology have been developed more quickly than in any other country in the world (Lee, O'Keefe, & Yun, 2003). One of the venues for educational reforms is, therefore, adopting and highlighting HOT, a core competency at various school levels in South Korea. For this study, the authors sought to develop a valid and reliable but pedestrian scale to measure the HOT that South Korean college students can use for their monitoring. In developing the scale, they assumed HOT skills to be a domain-general competency and a core competency for college students in current and future educational settings based on the previous literature (Amabile, 1996; Csikszentmihalyi, 1999; Ennis, 1962; Newmann, 1988; Resnick & Resnick, 1992; Siegel, 1992). Before development of the HOT skills scale, they elicited a definition and a factor structure of HOT skills from previous studies.

Carroll, Kathleen A.; Harris, Carolynn M (2021) explained the ability to synthesize and communicate complicated problems with confidence is paramount for success in STEM fields, but programs tend to overemphasize memorization. We conducted a preliminary study to determine if asking students to repeatedly link materials: (1) improved students' ability to build connections; and (2) increased students' confidence. Student accuracy and complexity response scores increased over the 5-week experiment, but student confidence did not. Our findings indicate the importance of providing students across STEM disciplines with as many opportunities as possible to practice using higher-order thinking skills to the benefit of STEM fields, educational programs, and students.

Dowd, Timothy J (2021) discussed higher-order thinking has been promoted in the American classroom for several decades. Summative assessments have the dual role of ascertaining the student's proficiency and also providing them with opportunities to employ various cognitive behaviors and to offer them with opportunities to demonstrate their ability to think in complex ways. The creators of one such summative assessment, the North Carolina End-of-Course English II Test, claim that their test provides up to 40% of its items at the higher-order of thinking. The purpose of this convergent mixed-methods study was to investigate how the language found in the released items on the North Carolina End-of-Course English II (Grade 10) Assessment compares with the language that promotes higher-order thinking found in the research literature. The qualitative component of this study focused on analyzing the language found in the questions and items on the NCEOC English II Assessment and how that language compared with the language used in the Hess Cognitive Rigor Matrix and the Webb Alignment Tool. The quantitative component of this student involved determining the frequency and percentage of assessment items that promote higher-order thinking. Four consultant coders, employing a double-rater read-behind consensus model to ensure inter-rater reliability, coded each of the 53 released items from the assessment according to its cognitive behavior and its thinking complexity. The study found that an overwhelming majority of the items on the NCEOC English II Assessment did not promote higher-order thinking.

Eko Andy Purnomo (2022) discussed that Problem-solving is the essence of mathematics and is the main goal in learning mathematics. Many students did not have good problem-solving skills based on the field observations. The problems grew up because the students were not used to solving the problems and the problem-solving stages. They did not include issues with high complexity, such as questions with the High Order Thinking Skills (HOTS) category. The problem-solving steps have been developed, such as; Dewey (1910), Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), and Wilson et al. (1993). The objective of this current study was to analyze the stages of problem-solving in solving mathematical problems with the High Order Thinking Skills (HOTS) category. The research sample was 57 students of Mathematics Education in one of Private University of Indonesia who took the Integral Calculus subject. This research was a qualitative descriptive study. The data analysis employed an inductive approach where the conclusions were drawn from minor case investigations to provide comprehensive results. The data analysis consisted of reduction data, presentation data, and concluding. There are three results from the current research. First, the students have not implemented problem-solving with problem-solving stages; second, the students fail to solve the problems due to a lack of mathematical literacy skills; last. The incomplete mathematization process causes imperfect problem-solving. Based on the results, the research recommendation is to add stages of problem-solving with two steps: formulating the situation mathematically and understanding mathematical solutions in real life or problems.

Made Rai Suci (2022) Shanti and others describes that Higher-order thinking skills (HOT) are expected to be skills needed now and in the future. Many learning strategies are applied to increase HOT. However, how effective is this in improving students' HOT abilities? This study will examine how effective learning is to improve HOT thinking skills. We use meta-analysis research techniques to analyze the research quantitatively. We have selected 21 articles from 60 Google Scholars that use models, media, and science learning assessments to improve HOT. Heterogeneity analysis (trim-fill, funnel plot approach, and Z value calculation) was used to prove the absence of publication bias. Forest plot analysis showed an average increase in learning effectiveness of 0.32 on the moderate effect. It shows that the effectiveness of learning is supported by strategies for using media, methods, and strategies for assessment. These factors are proven to increase the effectiveness of students' HOT abilities.

Richeal P. T. Kim How , Hutkemri Zulnaidi, and Suzieleez S. B. Abdul Rahim (2022) investigated that the Higher-Order Thinking Skill Test Instrument on Quadratic Equations (HOTS-QE) was developed to measure the level of HOTS on the topic of quadratic equations among secondary school students. The HOTS-QE instrument consists of 17 structured subjective items divided into two smaller questions according to the three levels of HOTS, namely applying, analysing, and evaluating. This study proposes a design and development research (DDR) approach via a development research that was done through an organised and systematic process. The DDR-oriented development process comprised seven stages namely source analysis, theory, development, content validity, validation of the HOTS domain accuracy, language validity, and pilot study. The development of the HOTS-QE instrument comprised seven stages, namely source analysis, theory, development, content validity, validation of the HOTS domain accuracy, language validity, and pilot study. The content validity of the instrument was evaluated by five mathematics education experts. The results showed that the item content validity index (I-CVI) value of each item was above 0.70 while the content validation per scale (S-CVI) value was 0.98. Results from the interrater evaluation also showed that the HOTS level accuracy of the instrument items had achieved the Cohen's Kappa coefficient value of 0.63. In addition, findings from the pilot study also showed that the Cronbach's alpha coefficient value was 0.79, the discrimination index value of each item was between 31.11% to 66.67%, and the difficulty index values were between 40.74% to 70.00%. These results suggest that the HOTS-QE instrument has an acceptable level of validity. The development of this instrument provides a more varied learning assessment that can foster students' interests in learning, help them to improve HOTS, and provide opportunities for students to directly apply the knowledge of quadratic equations learned at school in their daily life.

Keywords: Test instrument, Higher-Order Thinking Skills (HOTS), quadratic equations.

Nurulwahida Azid (2022) investigates the effect of incorporating higher order thinking skills (HOTS) in a school-based assessment (SBA) on mathematics achievement among students. A mixed-method case study design was used to assess a sample size of 158 students and three mathematics teachers. The students' proficiency in mathematics was determined by using the standard based curriculum for secondary schools. Mathematics assessment paper that was assiduously organized by incorporating HOTS questions. The students' thoughts on the incorporation of HOTS in the SBA were obtained through one-to-one face-to-face semi-structured interviews. The quantitative findings, which were analyzed using descriptive and regression tests, showed that 11.9% of the achieved mathematics score was contributed by incorporating HOTS in SBA, while 7.7% was contributed by SBA mastery level in mathematics. Students' maturity and misconceptions about math are among the challenges in implementing HOTS in SBA. The challenges in applying HOTS in SBA had a positive effect on teachers' pedagogical approach in a sense that they could devise a new strategy for meeting the needs of students and teach topics in alternative ways.

Lo, C. Owen; Feng, Li-Chuan (2020) examined the effects of higher order thinking skills (HOTS) interventions with gifted students in Taiwan. A total of 25 studies published between 1997 and 2017 were included. Twenty-nine effect sizes were extracted for the 25 studies. The small number of existing studies indicates a lack of scholarly attention to HOTS in gifted education in Taiwan in the past two decades. On the other hand, the effect sizes, ranged from 0.26 to 2.01, with a mean of 0.78 and standard deviation of 0.39, showed moderately large effect sizes for these interventions, which can be interpreted as evidence for general effectiveness. Subgroup analyses indicated that intervention effects did not vary significantly by grade level, type of program, intervention dosage, and type of dissemination. However, a statistically significant difference was found between the effect sizes in different types of instructional design (i.e. stand-alone HOTS unit vs. integrated HOTS unit). Implications are discussed.

Tsaparlis, Georgios (2020) analysed students' failure in the 2019 Nationwide Chemistry Examination in Greece, which concerns secondary education graduates, competing for admission to higher education Greek institutions. The distinction of thinking skills into higher and lower order (HOTS and LOTS) is used as a theoretical tool for this analysis. The examination included several questions that contained HOTS elements that had been unusual in previous examinations. This led to a decrease in overall student performance but better discrimination between outstanding and good students. Based on two samples of examination papers, corresponding to very similar subsets of the student population, the 2018 and 2019 examinations are compared, and the individual 2019 questions are evaluated. It was found that section B of the 2019 examination paper (which included contexts unfamiliar to the students, and for which, a large effect size between 2018 and 2019 was calculated) may have caused the large drop. An important link is established between the 2019 low performance and the HOTS and LOTS features of the questions, and the role or non-role of algorithmic calculations is examined. In addition, the critical opinions of chemistry teachers are provided, with a consensus emerging in favour of connecting chemistry with everyday life.

Debbie Marie B. Verzosa and others (2021) describes that Mobile technology can provide a potential solution, especially when application (app) design is based on sound pedagogical principles and gamification elements. However, an inventory of available mobile apps for mathematics reveals that many of the available apps are guided by a behaviorist perspective that favors repetition over meaningful learning. This paper reports on the design of mobile mathematics apps that harness gamification techniques to promote higher-order thinking skills (HOTS) even in basic elementary school concepts such as number comparison, and addition and subtraction. The integration of these apps in the classroom is also discussed.

Johan Setiawan, Ajat Sudrajat, Aman, Dyah Kumalasari (2021) described that 1) Produce higher order thinking skill (HOTS) assessment instruments in learning Indonesian history; 2) Know the validity of HOTS assessment instruments in learning Indonesian history; 3) Find out the characteristics of HOTS questions in learning Indonesian history. This study employed the research and development method of the Borg and Gall model. The HOTS test item was conducted on 36 students in class XI of 2 Ngaglik State Senior High School. Data analysis includes tests of validity, reliability, level of difficulty, distinguishing features and deception index. The study found: 1) The HOTS assessment instrument of multiple-choice questions consisted of 25 items; 2) The results of the HOTS question validation by two Indonesian history learning assessment experts on the material, construction and language aspects were valid and appropriate. The results of the validation by three Indonesian history teachers also stated that the assessment instruments were valid and appropriate; 3) The characteristics of HOTS questions had fulfilled the validity criteria of 23 questions, reliability with a coefficient of 0.97 (very strong), the average difficulty level is 0.33 (moderate), the average differentiation test is 0.42 (good), and the average deception index is 0.56 (good).

Nara Hari Acharya (2021) explored that mathematics teachers' perception on Higher Order Thinking Skills (HOTS). The explanatory sequential mixed method with Likert scale and interview guideline was used as research tools for data collection. 50 mathematics teachers who were teaching at Higher Education were conveniently selected for the survey and 5 of them were interviewed. The mean and standard deviation of different views on Likert scale were calculated and the results from quantitative data are presented in language with the help of qualitative data obtained from the interview. The teachers' perception was gathered about concept or understanding, needs, clarity, and practice about the HOTS. The study found that most of the teachers viewed HOTS as a commonly known idea of analyzing and synthesizing skills together with logical thinking and decision-making skills. In depth, teachers were clear about the meanings, strategies and the use of HOTS but weak in implementation. Majority of the teachers viewed the practice of HOTS in mathematics classrooms as necessary but they were rarely used. Only a few of them were partially practicing them in classroom instruction. The teachers felt complexity in practicing HOTS due to students' basic knowledge, approach and access to different materials, teachers' training, curriculum and time of implementation in development of HOTS in students.

2.2 CONCLUSION

We elaborately discussed various studies related with Student's Ability in Solving Higher Order Thinking Skills (HOTs) Mathematics Problem Based on Learning Achievement in this chapter.

CHAPTER III

THE PRESENT STUDY

3.01 INTRODUCTION

Mathematics is essential for the individual to learn. According to Chen mathematics is very important for various fields with real-life applications, including natural sciences, engineering, medicine, and social sciences. Mathematics can be used to develop skills that involve logical, systematic, critical, careful and creative reasoning skills in communicating ideas or solving problems. Mathematics problems are useful for training students to reflect and analyze mathematics. Therefore, students must be taught how to solve problems with appropriate problem-solving strategies.

Problem-solving is an essential part of mathematics. Problem-solving is one way to allow students to develop a deeper understanding of mathematical concepts, English, and various ways to represent mathematical solutions. Students must be encouraged to reflect on their thoughts during the problem-solving process so that they can implement, adjust and modify appropriate strategies to find solutions, students need opportunities to formulate, solve, and solve complex problems.

3.02 STATEMENT OF THE PROBLEM

The present study is termed "**STUDENTS' ABILITY IN SOLVING HIGHER ORDER THINKING MATHEMATICS PROBLEM BASED ON LEARNING ACHIEVEMENT – A STUDY**".

3.03 NEED AND SIGNIFICANCE OF THE STUDY

Educational systems worldwide underscore the importance of developing higher-order thinking skills (HOTS) to prepare students for the new challenges of the 21st century. Some pressing issues faced by educators include the ambiguity of the construct; the implementation of HOTS in classroom practices; and the implications for teaching students from linguistically and culturally diverse backgrounds.

We observed that in the National Achievement Survey 2021, The state average of Tamil Nadu in 10th standard in mathematics was 56, which is less than the National average (57). we also find that there are 12 learning outcomes taken for assessment and find the highest average and lowest average in mathematics are 34 and 21 respectively.

We have observed that the performance of 10th-standard students in mathematics is not adequate. If the word problem is given then students are able to understand the given problem, translate the given problem in their own words, select appropriate strategies, solve by using strategies and also review the solution. Especially, if you make any changes in the textbook problem then, students in high achievers also have been find difficulty in solving the given problems. This study mainly focused on low and high achiever's how they differ in ability to solve higher-order thinking problems. From these necessities, we need to understand how they faced difficulties in solving problems, especially HOT sums.

HOTS assessment to measure students' abilities in the field of Mathematics and Science internationally has been carried out by Trends in the International Mathematics and Science Study (TIMSS) and the Program of International Student Assessment (PISA). Based on the results of the PISA study in 2009, India was ranked 72 out of 73 participating countries. Therefore, to improve the quality of Indian education, especially in the field of mathematics which refers to international education, it can be done by training students' skills in solving HOTS mathematical problems.

3.04 OPERATIONAL DEFINITION

3.04.01 MATHEMATICS PROBLEM:

A mathematics problem is a question or situation that requires mathematical thinking and problem-solving skills to find a solution. These problems can vary widely in complexity, from simple arithmetic calculations to intricate puzzles that involve advanced mathematical concepts.

3.04.02 PROBLEM SOLVING

Problem-solving is an essential part of mathematics. Problem-solving is one way to allow students to develop a deeper understanding of mathematical concepts and various ways to represent mathematical solutions. Problem-solving refers to the process of finding solutions to difficult or complex issues or challenges. It involves identifying a problem, understanding its root causes, brainstorming potential solutions, evaluating those solutions, and implementing the best one.

3.04.03 HIGHER ORDER THINKING (HOT)

Higher-order thinking is not a new concept and it concerns synthesis, evaluation, interpretation, hypothesizing, prediction, conjecture, critical thinking and judgement. It is complex and involves reflection, self-regulation, testing of ideas, and problem-solving. In Higher order thinking mathematical problem is a non-routine mathematical problem that contains elements of analysis, evaluation, and creation.

3.04.04 LEARNING ACHIEVEMENT

Learning Achievement or Academic achievement is the outcome of education, the extent to which a student has achieved their educational goals. Learning Achievement is commonly measured by examinations or continuous assessment.

The half-yearly exam scores of 10th students in Mathematics subjects have been taken into account for the measurement of the academic performance of the student.

3.05 OBJECTIVES OF THE STUDY

- To assess the ability of students to solve Higher Order Thinking (HOT) mathematics problems with high and low learning achievement based on Krulik and Rudnick's problem-solving.
- To find the relationship between the ability of students to solve Higher Order Thinking (HOT) mathematics problems and learning achievement.

3.06 RESEARCH QUESTIONS

1. What are the Abilities of Students to Solve Higher Order Thinking Mathematics Problems on Learning Achievement?

2. What are the abilities of students to solve Higher Order Thinking Skills (HOTS) mathematics problems among high and low learning achievement based on Krulik and Rudnick's problem-solving?
3. Why do students lack in solving HOT Mathematics Problems?
4. Is there any correlation between students' Ability to solve Higher Order Thinking Mathematics Problems and their Learning Achievement?

3.07 METHOD OF THE STUDY

This research uses the **Descriptive, Mixed method (both qualitative and quantitative methods)** in the form of case studies. The results of the study illustrate the ability of students to solve mathematical problems of Higher Order Thinking (HOT) based on student learning outcomes. The Sample was chosen by the researcher using **Purposive - Stratified Sampling**. In this 3 best performance Government High Secondary Schools in the Ranipet district were chosen. In that school, there are eight students from the 10th standard were chosen based on the results of the Half Yearly Examination score in mathematics, from these 8 students, were 4 students from low achievers and 4 students from high achievers were selected.

3.08 TOOL USED IN THE STUDY

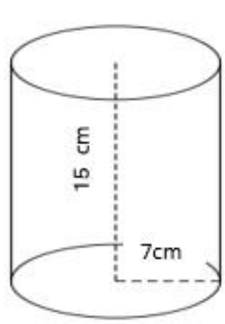
This research includes a Mathematics question paper and an interview schedule. we conducted a workshop for preparing the test items and also the interview schedule of the 10th standard.

The mathematics question paper in this study consists of 5 Higher Order Thinking Skill-based word problems. These test questions were prepared by the Researcher with the elements of Revised Bloom's Taxonomy questions like analysis, evaluation, creation and also which were not routine mathematical problems. It is shown below:

TABLE 3.1 MATHEMATICS QUESTION PAPER

Class: 10 Subject: Mathematics

SL. NO	UNIT NAME	SUBTOPIC	QUESTION
1.	Unit 1 **Relations and Functions**	Exercise 1.4 Similar problem 12	f என்ற சார்பானது செல்சியஸில் (C) உள்ள வெப்பநிலையையும், பாரன்ஹீட்டில் (F) உள்ள வெப்பநிலையையும் இணைக்கும் சார்பாகும். மேலும் அது f(C) = F என வரையறுக்கப்பட்டால், (இங்கு, F = (9/5) x C + 32) செல்சியஸ் மதிப்பும் பாரன்ஹீட் மதிப்பும் சமமாக இருக்கும் பொழுது வெப்பநிலை ஆகியவற்றைக் கண்டறிக.
2.	Unit 2 **Numbers and Sequence**	Exercise 2.7 Similar problem 11	மதன் ஒரு பணிக்கான நேர்காணலில் பங்கேற்கிறார். அந்நிறுவனம் அவருக்கு இரண்டு வாய்ப்புகளை வழங்குகிறது. வாய்ப்பு A: முதல் மாத ஊதியம் ரூ. 20,000 மற்றும் நிச்சயமான 10% ஆண்டு ஊதிய உயர்வு 5 ஆண்டுகளுக்கு.

			வாய்ப்பு B: முதல் மாத ஊதியம் ரூ. 24,000 மற்றும் நிச்சயமான 5% ஆண்டு ஊதிய உயர்வு 5 ஆண்டுகளுக்கு. A மற்றும் B ஆகிய இரு வாய்ப்புகளில் 10 வது வருட ஊதியத்தில் a) எது சிறந்த வாய்ப்பாக இருக்கும்? b) அவற்றிற்கு இடையிலான வேறுபாடு எவ்வளவு?
3.	Unit 2 Numbers and Sequence	Exercise 2.9 Similar problem 6	அனிதா தன்னிடமுள்ள 6 அடி, 7 அடி, 8 அடி ....., 15 அடி ஆகியவற்றை பக்க அளவாக கொண்ட 10 சதுர வடிவ வண்ணக் காகிதங்கள் இருந்தன. அவற்றைக் கொண்டு 1200 சதுர அடி பரப்பளவை அலங்கரிக்க நினைக்கிறார். இது சாத்தியமா? என ஆராய்க.
4.	Unit 4 Geometry - 4.2.2 Criteria of Similarity, Page 171	Similar problem of Example 4.4	100 செ.மீ உயரமுள்ள ஒரு சிறுவன் விளக்கு கம்பத்தின் அடியிலிருந்து 1.5 மீ/வினாடி வேகத்தில் நடந்து செல்கிறான். தரையிலிருந்து விளக்கு கம்பத்தின் உயரம் 4 மீ எனில், 6 வினாடிகள் கழித்துச் சிறுவனுடைய நிழலின் நீளத்தைக் காண்க.
5.	Unit 7 Mensuration	7.3 Volume - Volume of frustum of a Cone	கொள்கலன்கள் A மற்றும் B ஐப் படத்தில் காணலாம். 

			A **B** a) எந்த கொள்கலனின் கொள்ளளவு அதிகம்? b) கொள்கலன் A மற்றும் B ஆகியவற்றின் கொள்ளளவுகளின் விகிதம் காண்க.

The results of the test are students' ability to solve HOTS problems analyzed based on Krulik and Rudnick's problem-solving. We prepared an interview schedule based (See in the annexure) on Krulik and Rudnick's problem-solving steps. Hence, the selected student interviewed with an in-depth question based on their response in solving HOTS problems given in the test. The responses of selected students in each test question were interviewed with in-depth questions based on Krulik and Rudnick's problem-solving steps.

3.09 RELIABILITY AND VALIDITY OF THE TOOL

Reliability refers to the accuracy of measurement by a tool. For the present tool, the coefficient of internal consistency is 0.76 by the Cronbach's alpha method. The coefficient of stability is 0.72 by the test re-test method. Hence the tool is reliable.

Validity is the extent to which a test measures what it intends to measure. The face validity of the question paper has been established beyond doubt that the question paper reflects the various areas that students encounter in solving HOT-based problems. The question paper has construct validity, as the items were selected using the ‘t’ value according to Edward L. Allen.

The intrinsic validity of the question paper was found to be 0.87 by taking the square root of the reliability coefficient. To find out its content validity, the question paper was given to experts, and they agreed that the items in the scale provided adequate coverage of the concept.

3.10 SAMPLE AND SAMPLING TECHNIQUE

The population of the present study was tenth-standard students in Ranipet District. In this study, the researcher used the Purposive – Stratified Sampling Technique to select the schools. Also, the researcher has chosen 3 schools which best perform in the 10th public exam in the Ranipet District. The sampled schools are as follows:

Sl. No	Name of the School
1	GHSS, Kodaikkal
2	GGHSS, Walaja
3	GGHSS, Arcot

Based on learning achievement, four students with the highest learning achievement were taken and four students with low learning achievement in the three schools.

TABLE 3.2 MATHEMATICS SCORE OF STUDENTS IN HALF-YEARLY EXAM

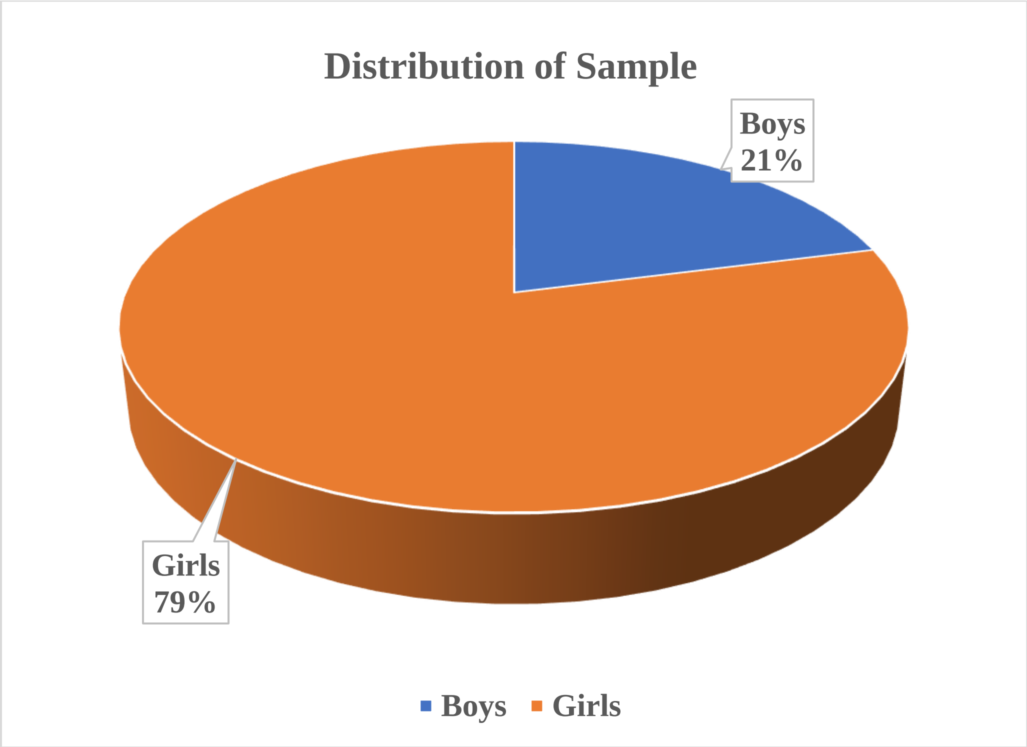
Sl. No	Name of the Student	Name of the School	Mathematics Score in Half Yearly Exam
1	Y. Astin	GHSS, Kodaikkal	30
2	R. Devendiran	GHSS, Kodaikkal	35
3	S. Sowntharya	GHSS, Kodaikkal	35
4	S. Saravanan	GHSS, Kodaikkal	36
5	C. Gokulakrishnan	GHSS, Kodaikkal	93
6	S. Shalini	GHSS, Kodaikkal	77
7	J. Gokulakrishnan	GHSS, Kodaikkal	93
8	B. Hemavarshini	GHSS, Kodaikkal	93
9	K Kumudhavalli	GGHSS, Walaja	23
10	S Lakshmi	GGHSS, Walaja	26
11	S Ayusha	GGHSS, Walaja	40
12	M Sudharani	GGHSS, Walaja	42
13	J Vaishnavi	GGHSS, Walaja	80
14	S. Pavithra	GGHSS, Walaja	81
15	R. Yamuna	GGHSS, Walaja	85
16	M. Arunasri	GGHSS, Walaja	89
17	S. Shalini	GGHSS, Arcot	26
18	S. Keerthana	GGHSS, Arcot	35
19	I. Athiyabegum	GGHSS, Arcot	40
20	J. Rasiya Sulthana	GGHSS, Arcot	41
21	V. Prithika	GGHSS, Arcot	82
22	P. Gayathri Sri	GGHSS, Arcot	85
23	S. Ilakkiya	GGHSS, Arcot	98
24	S. Kaviya	GGHSS, Arcot	98

TABLE 3.3 DISTRIBUTION OF SAMPLE

S. No	Variables	Categories	Frequency	Percentage

1	Gender	Boys	5	21%
		Girls	19	79%

FIGURE 3.1 DISTRIBUTION OF SAMPLE



3.11 ADMINISTRATION OF TOOL

The 3 best performance schools were selected in the Ranipet district by the researcher consulted with the chief Educational Office. From these schools, the selected students allowed by the researcher was respond to the given HOT questions in the test. The time given to students to complete the test was around 30 minutes. After completing the test, each student was interviewed by the researcher using an interview schedule.

3.12 SCORING DATA

The test consists of 5 HOT questions. After getting the responses of students, each question was assessed in a score interval of 0 to 5 by the researcher. The highest and lowest scores were 5 and 0 respectively.

After assessing the responses to given questions, each student was interviewed with the following questions through an interview schedule based on Krulik and Rudnick's problem-solving steps:

- Whether students have recognized where is the problem coming from?
- Whether students understand the given problem or not?
- Whether each student chooses an appropriate strategy for the given problem or not?
- Whether each student solves the problem using strategies?
- Whether each student verifies the answer to the problem?

3.12 STATISTICAL TECHNIQUES USED FOR DATA ANALYSIS

In the present study following statistical techniques were used.

1. Descriptive Analysis (Mean, Standard Deviation)
2. Differential Analysis (t-value and F-ratio)
3. Correlation Analysis (r-value)

3.13 DELIMITATION

1. Geographically, the sample area selected was Ranipet District of Tamil Nadu.
2. The three best higher secondary schools in Ranipet District were selected through Purposive – Stratified Random Sampling for this study.
3. This study was restricted to tenth-class students in Rural and Urban Areas.

3.14 STRUCTURE OF THE REPORT

In Chapter I, Meaning of Problem, Mathematics Problem, Problem Solving, Steps in Effective Problem Solving, Stages of Problem-Solving Krulik and Rudnick, Monitor and Reflect on the Process of Mathematical Problem Solving, Categories of the Cognitive Process Dimension in B. S. Bloom Taxonomy and also High Order Thinking Skills Based on Bloom's Taxonomy of Educational Objectives were discussed in details.

In Chapter II, Review of related studies, the investigations, similar to the present study, have been summarized and presented in the form of abstracts.

In Chapter III, the Statement of the Problem, Need and Significance of the Study, Operational Definition, Objectives of the Study, Method of the

Study, Tool Used in the Study, Sample and Sampling Technique have been discussed.

The details regarding analysis of the data, results of statistical analyses and the interpretation of results are described in Chapter IV.

Chapter V represents a summary of the study, major findings, conclusions, educational implications of the study and suggestions for further research.

The bibliography is given following the chapter V. The appendices containing a copy of the tool is included next to the bibliography.

3.15 CONCLUSION

This chapter outlines the design of the present study, the procedure or method of study followed and the nature of the sample and sampling technique used. It also describes the research questions to be tested, the tools used, and the method of administration and scoring.

CHAPTER IV

ANALYSIS AND INTERPRETATION OF DATA

4.01 INTRODUCTION

The data collected about student's ability to solve higher order thinking skills (HOTS) mathematics problems based on learning achievement along with personal variables were analyzed concerning the objectives and research questions of the study. The data have been subjected to statistical analysis namely, Descriptive, Differential and Correlation Analysis. The result of statistical analyses has been summarized along with interpretation in this chapter.

4.02 RESEARCH QUESTION 1:

What are the Abilities of Students to Solve Higher Order Thinking Mathematics Problems on Learning Achievement based on Krulik and Rudnick's Problem-Solving?

Result:

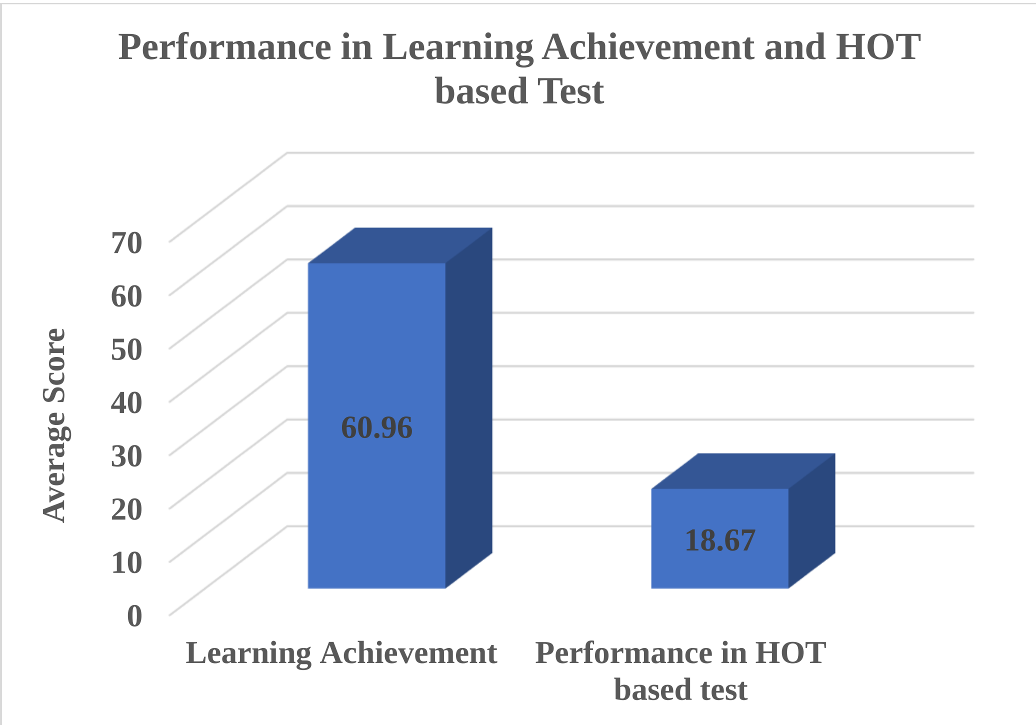
Students' learning achievement in the half-yearly exam and the test, students' ability in solving Higher Order Thinking Skills mathematics problems based on Krulik and Rudnick's Problem Solving calculated the same are given in the table.

TABLE 4.1 STUDENTS' SCORES IN HALF-YEARLY EXAM AND HOT-BASED TEST

Average score of Learning Achievement	Average Performance in HOT based test
60.96	**18.67**

In the table 4.1 shows that the average score of Learning Achievement in the half-yearly exam and Average Performance in solving HOT-based tests are 60.96 and 18.67 respectively. High and Low learning achievers in the same class were chosen by the Researcher and their average score of Learning Achievement is 60.96 which is a moderate performance in the half-yearly exam. But in HOT based test, their performance (18.67) was too bad.

FIGURE 4.1 STUDENTS' SCORE IN HALF YEARLY EXAM AND HOT BASED TEST



4.03 RESEARCH QUESTION 2:

What are the abilities of students to solve Higher Order Thinking mathematics problems between high and low learning achiever based on Krulik and Rudnick's problem-solving?

Result:

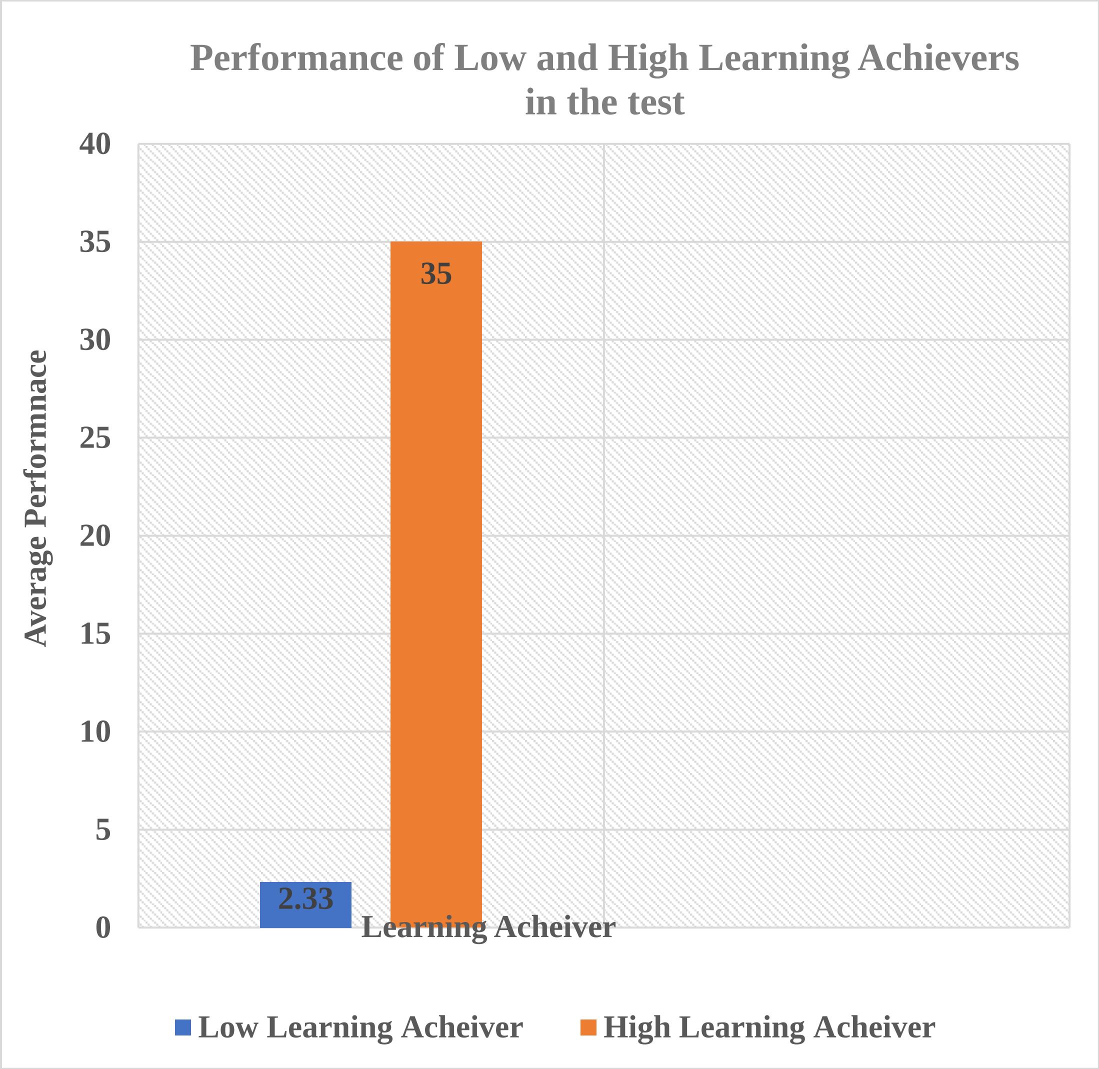
The performance of high and low learning achievers in Learning achievement and solving Higher Order Thinking mathematics problems in the test was calculated and given in the table.

TABLE 4.2 SCORES OF LOW AND HIGH ACHIEVEMENTS STUDENTS

Learning Achiever	Average Performance in the half-yearly exam	Average Performance in the test based on Krulik and Rudnick's problem-solving
Low	34.08	2.33
High	87.83	35

In the table 4.2 shows that the average achievement of the Low and High Achiever in the half yearly exam are 34.08 and 87.83 respectively. Also, we found that the test result of HOT based test, the Average performance of the Low and High Achiever are 2.33 and 35 respectively.

FIGURE 4.2 SCORES OF LOW AND HIGH ACHIEVEMENTS STUDENTS



From the above table 4.2 and graph, we found that the high-learning achievers (35) have better performance than the low-learning achievers (2.33). In this regard, low learning achievers have too less in their performance, because their understanding and recognition of the given HOT problems was very low.

Even though the average performance of high learning achievers (35) was not good enough it means that high-learning achievers did not perform well in solving HOT-based problems.

4.04 RESEARCH QUESTION 3:
WHY DO STUDENTS LACK IN SOLVING HOT MATHEMATICS PROBLEMS?

Question 1: The function 'f' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by f(C) = F where F = (9/5) x C + 32. Find the temperature when the Celsius value is equal to the Fahrenheit value.

Results of Question 1:

It is an analysis type question which is based on Bloom's Revised Taxonomy. The researcher expected students to understand the concept of function, apply the **given condition** to the problem and after that solve the equation and also verify the same.

The table shows how many students got the scores from 0 to 5 in percentage.

TABLE 4.3 STUDENTS SCORE FOR QUESTION 1

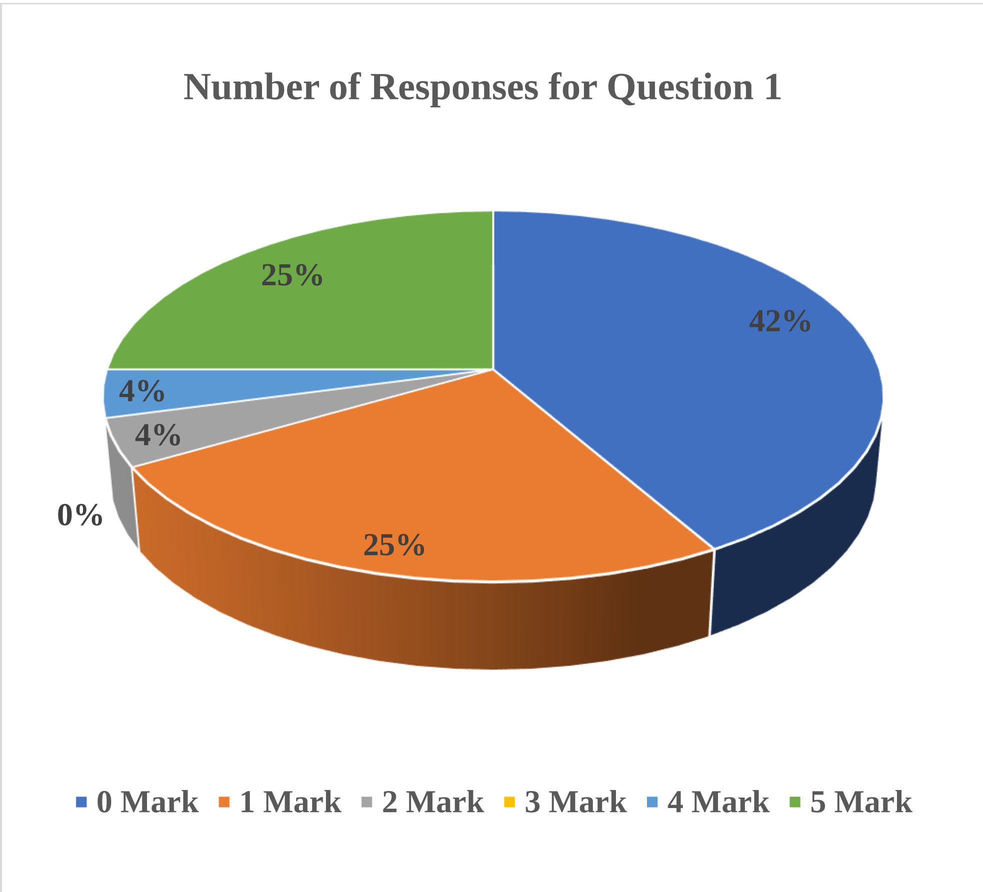
Marks	0 Mark	1 Mark	2 Mark	3 Mark	4 Mark	5 Mark
Number of Responses (%)	42%	25%	4%	0%	4%	25%

From the above table 4.3, we found that the following findings:

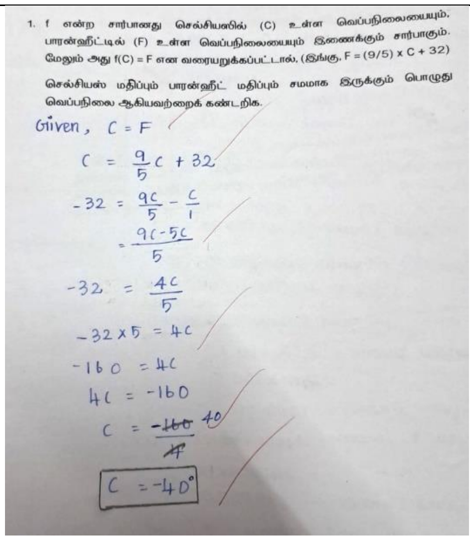
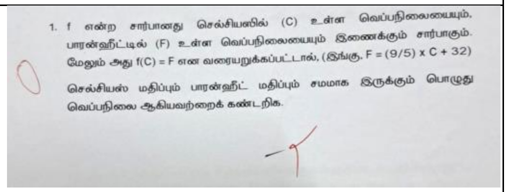
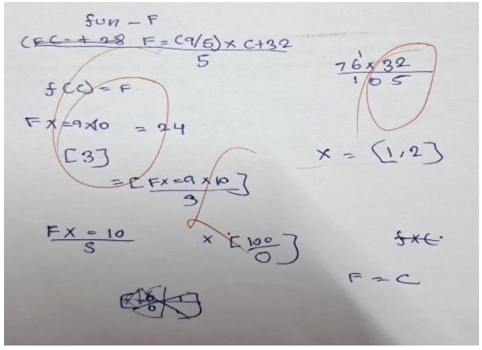
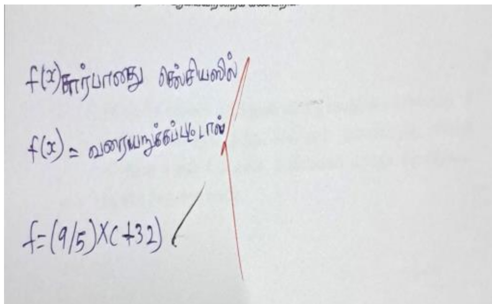
- In above question 1, 42% of students got 0 marks, 6% of students got 1 mark, 4% of students got 2 marks, 4% of students got 4 marks and only 25% of students got full marks.
- 10 students out of 24 students (42%) scored zero marks for the first question. Many students didn't know "where question 1 comes from" and they didn't understand the concept in which the topic or sub-topic was covered.

- 40% of students (only 6 out of 24 students) are unable to understand how to solve the equation of question 1.
- 71% of students (17 out of 24 students) scored less than 3 marks for question 1. We know that Students are unable to fully resolve to understand or find a solution.
- 54% of students (13 out of 24 students) are unable to find the method of solving for the given question 1.
- Only 25% of students (6 out of 24 students) were solved the question 1 completely. From this, it is clear that 75% of students are unable to solve question 1 completely.

FIGURE 4.3 STUDENTS RESPONSES OF QUESTION 1



Student's ability in solving HOTS mathematics problems with high and low learning achievement in item 1 is as follows:

Achievement	High learning achievement	Low learning achievement
The answers of Students with learning achievement		  
Reasons for their correct/wrong responses from the interview	Only 25% of students (6 out of 24 students) solved the question 1 completely. It reveals that they understood the problem, substituted the condition into a given function and solved the equation correctly.	• Many students didn't know "where question 1 comes from" and they didn't understand the concept in which topic or sub-topic was covered. • 42% of students did not know what to do and some of them wrote irrelevant answers (see in the above samples).

Question 2:

Madhan is attending an interview for a job and the company gave two offers to him.

Offer A: ₹ 20,000 to start with followed by a guaranteed annual increase of 10% for the first 5 years.

Offer B: ₹ 22,000 to start with followed by a guaranteed annual increase of 5% for the first 5 years.

a) What is his salary in the 10th year for offers A and B?

b) What is the difference between them?

Results of Question 2:

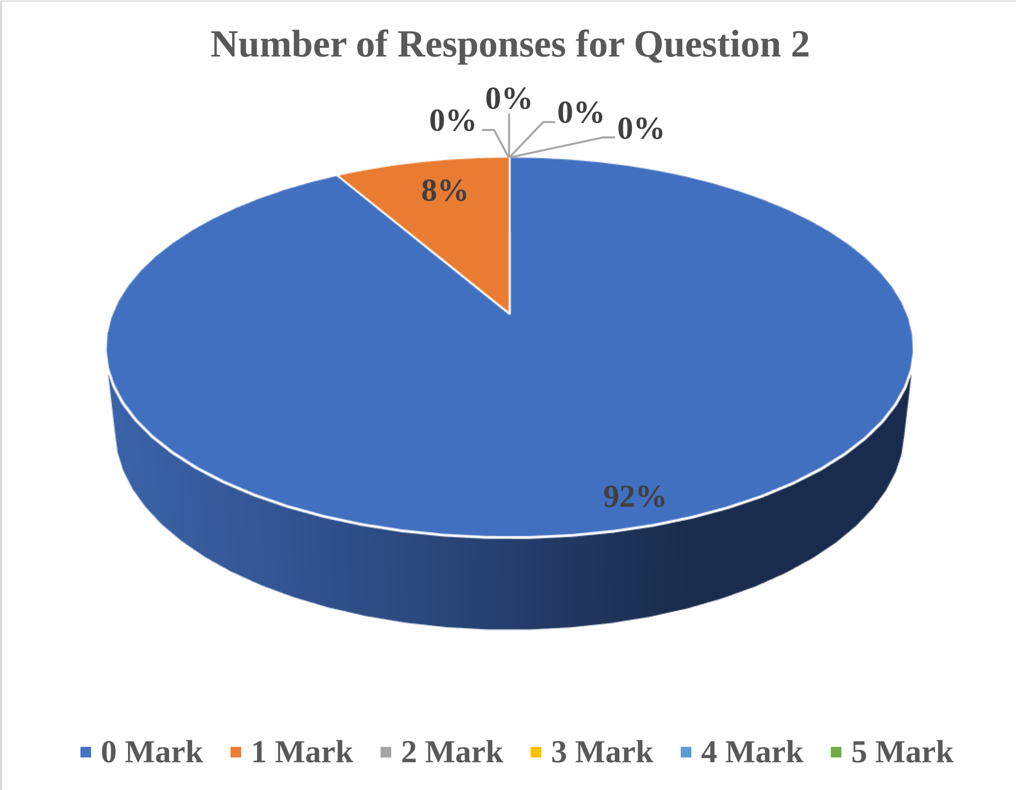
It is an analysis and judgmental thinking type question which is based on the Blooms Revised Taxonomy. The researcher expected students to select the formulae for **sum to n terms of an Arithmetic Progression**, substitute the given data and solve the problem and finally find the best offer from these twos.

The table shows how many students got the scores from 0 to 5 in percentage.

TABLE 4.4 STUDENT'S SCORE FOR QUESTION 2

Marks	0 Mark	1 Mark	2 Mark	3 Mark	4 Mark	5 Mark
Number of Responses (%)	92%	8%	0	0	0	0

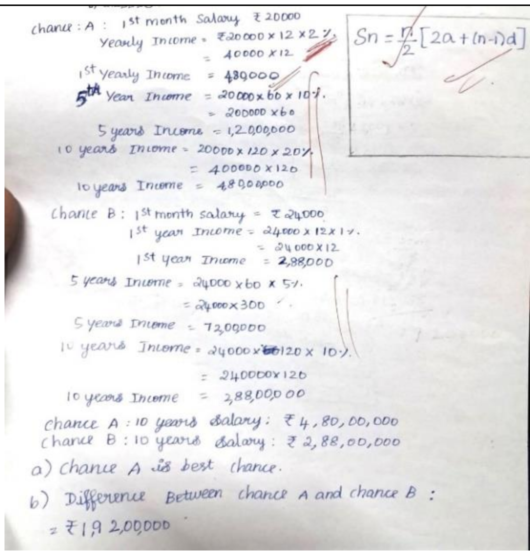
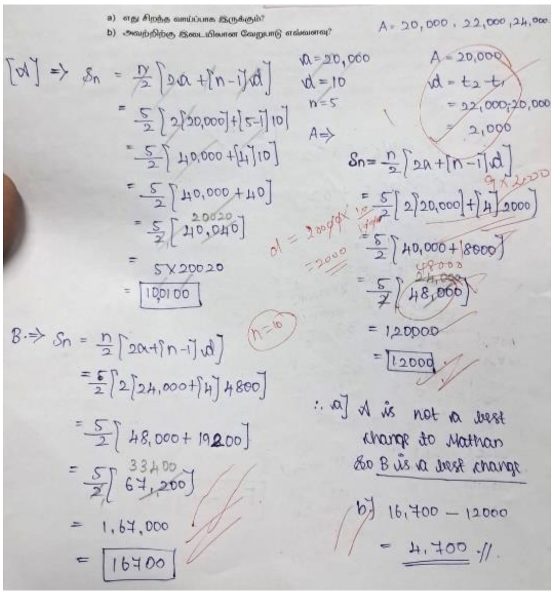
FIGURE 4.4 STUDENTS RESPONSES OF QUESTION 2

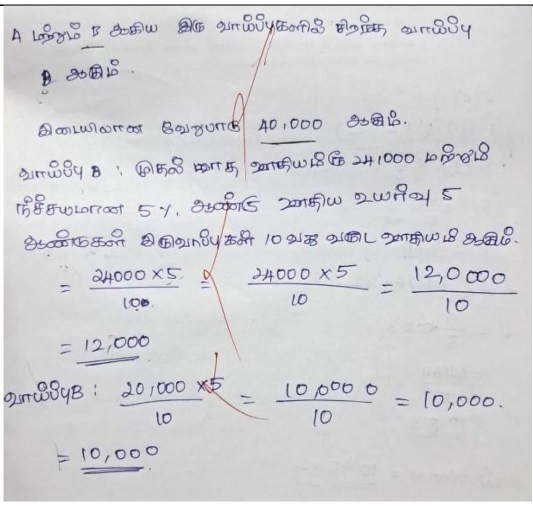


From the above table 4.4, we found that the following findings:

- All students are unable to solve question 2 completely. Many of them do not recognise that "the given question in which topic or subtopic is covered?". Also, they have not understood the question and may not find the relevance of the concepts.
- 8% of students only (2 students) were able to recognise that "the question belongs to which topics" and students also find that what is given, is what is to be found in the question.
- 92% of students did not get any marks, which means 92% of students are not experienced enough to approach question 2.
- Only 2 out of 24 students scored 1 mark for question 2.
- All the students did not select and apply the formula to solve the given problem.

Student's ability in solving HOTS mathematics problems with high and low learning achievement in item 2 is as follows:

Achievement	**High learning achievement**	**Low learning achievement**
The answers of Students with learning achievement	 	

		
Reasons for their correct/wrong responses from the interview	• All students are unable to solve question 2 completely. • 92% of students do not recognise that "the given question in which topic or subtopic is covered?". Also, they have not understood the question and may not find the relevance of the concepts. • Even high achievers did not choose the formulae/steps for solving. Low achievers did not respond anything for this question.	

Question 3:

Anitha has 10 square colour papers of sizes 6 cm, 7 cm, 8 cm, …, 15 cm. She wants to decorate a 1200-square-foot area with them.
Is this possible? Explore it.

Results of Question 3:

It is an Evaluation type question which is based on the Blooms Revised Taxonomy. The researcher expected students to find the formulae for the **sum of squares of first n natural numbers** substitute the given information and solve the problem.

The table shows how many students got the scores from 0 to 5 in percentage.

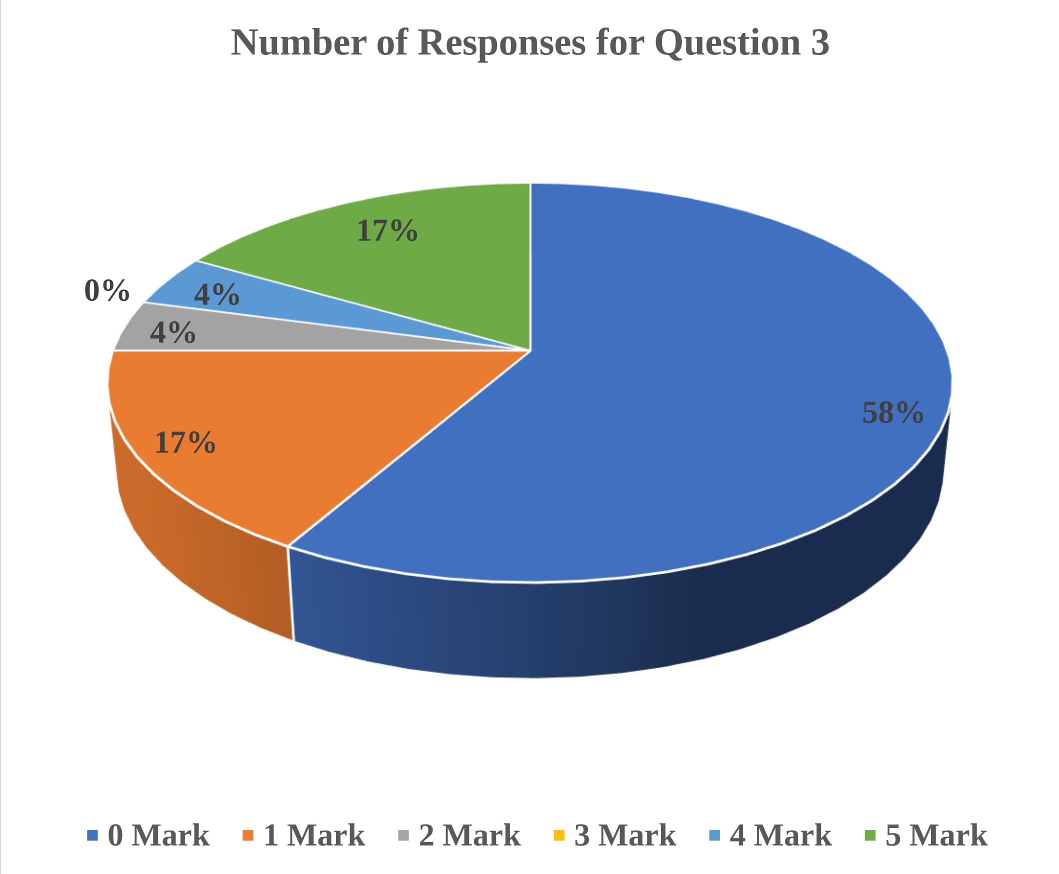
TABLE 4.5 STUDENT'S SCORE FOR QUESTION 3

Marks	0 Mark	1 Mark	2 Mark	3 Mark	4 Mark	5 Mark
Number of Responses (%)	58%	17%	4%	0%	4%	17%

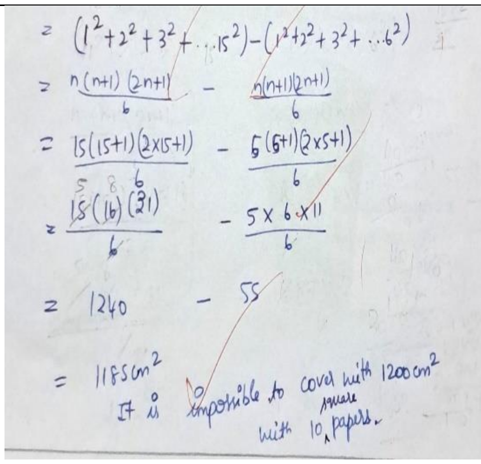
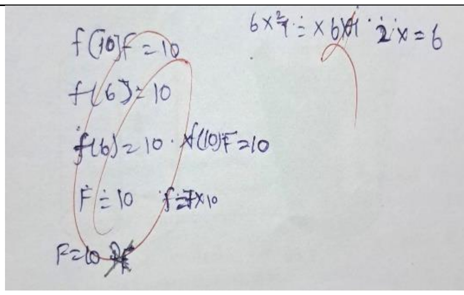
From the above table 4.5, we found that the following findings:

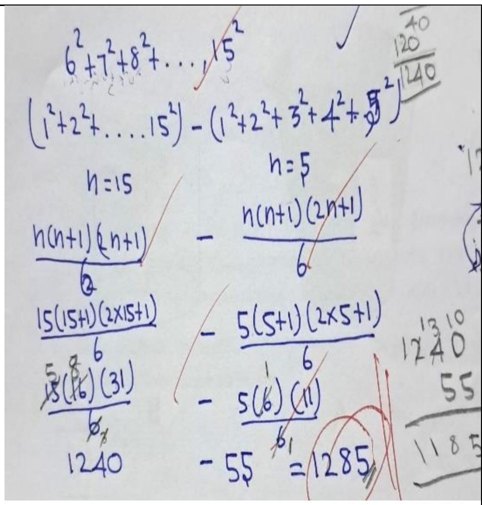
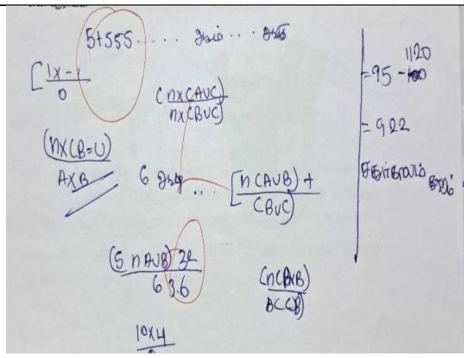
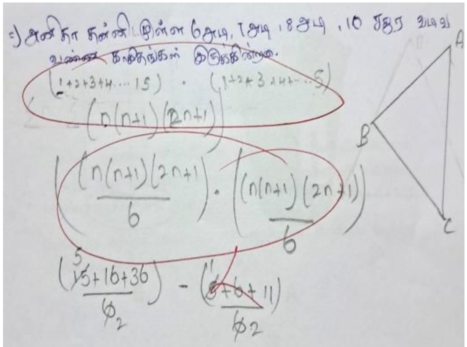
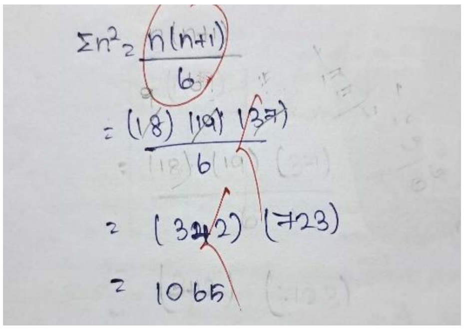
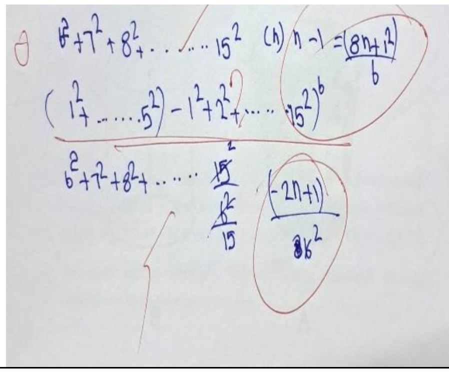
- 58% of students got 0 marks and 17% of students got 1 mark, so 75% of students were unable to understand, plan and solve question 3.
- Only 17% of students (4 out of 24 students) answered this question completely. So, it reveals that 83% of students did not answer question 3 completely.
- It reveals that 83% of students did not respond completely. They were unable to solve due to not understanding the given calculation properly, not knowing how to convert it into a mathematical statement, not knowing how to choose the optimal formula, and not using the formula properly.

FIGURE 4.5 STUDENTS RESPONSES OF QUESTION 3



Student’s ability in solving HOTS mathematics problems with high and low learning achievement in item 3 is as follows:

Achievement	**High learning achievement**	**Low learning achievement**
The answers of Students with learning achievement	 A complete response was given by the student	

	 A partial response was given by the student	 **Irrelevant Answers**   **Wrong formulae selection** 
Reasons for their correct/wrong	Students do find the formulae for the sum of squares of first n natural numbers substitute the	• 83% of Students answered irrelevantly or selected the wrong formulae.

responses from the interview	given information and solve the problem correctly.	They were unable to solve due to not understanding the given calculation properly, not knowing how to convert it into a mathematical statement, not knowing how to choose the optimal formula, and not using the formula properly.

Question 4:

A boy of height 100 cm is walking away from the base of a lamp post at a speed of 1.5 m/sec. If the lamppost is 4 m above the ground, find the length of his shadow cast after 6 seconds.

Results of Question 4:

It is an Assessment / Evaluation type question which is based on the Blooms Revised Taxonomy. The researcher expected students to convert the problem into a diagram, form an equation using the concept of congruency solve the equation and verify the same.

The table shows how many students got the scores from 0 to 5 in percentage.

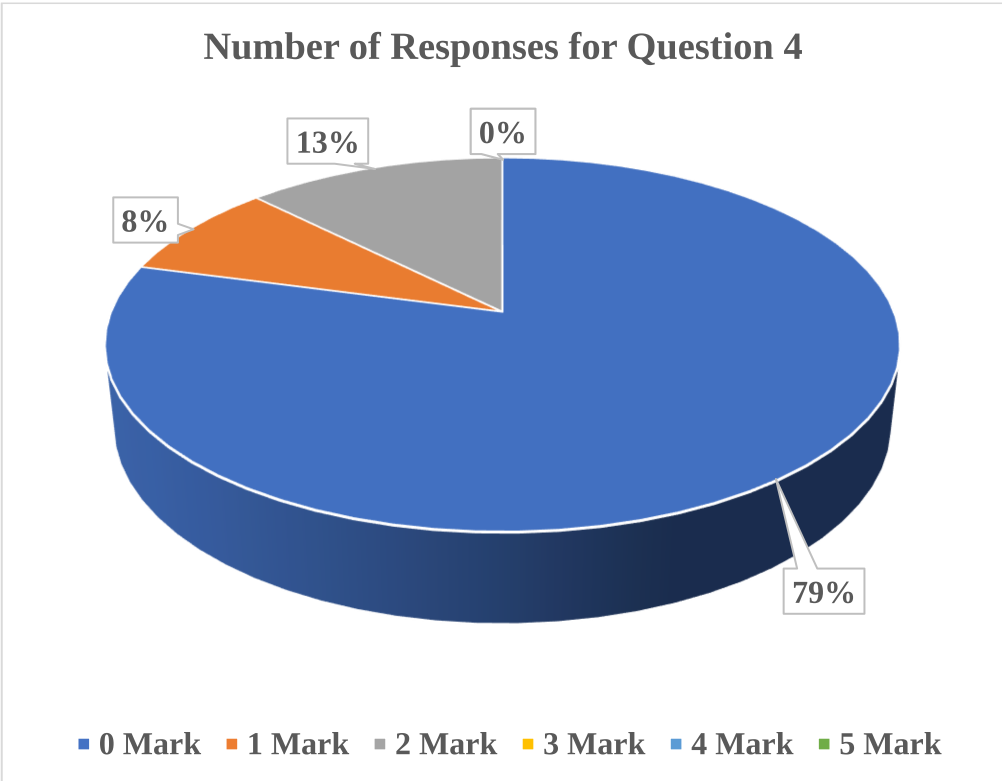
TABLE 4.6 STUDENT'S SCORE FOR QUESTION 4

Marks	0 Mark	1 Mark	2 Mark	3 Mark	4 Mark	5 Mark
Number of Responses (%)	79	8	13	0	0	0

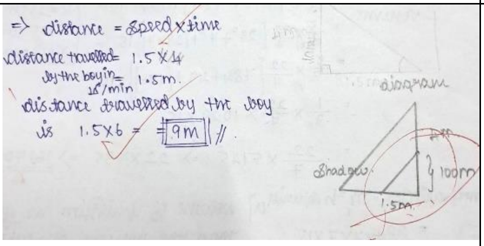
From the above table 4.6, we found that the following findings:

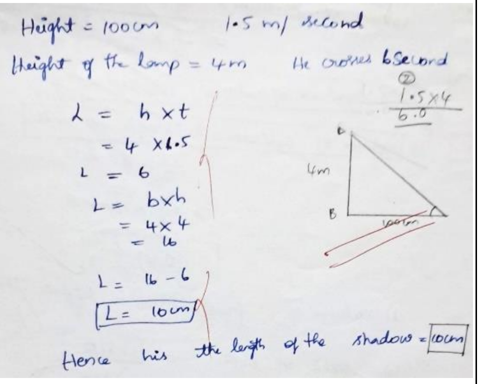
- 79% of students got 0 marks, 8% of students got 1 mark, and 13% of students got 2 marks so 100% of students did not solve question 4 completely. It reveals that most of the students could not recognize and understand the given problem.
- Question 4 was not solved completely by all the students. Students could not understand nor it be solved systematically.
- Each student scored less than 2 marks for question 4. That means they have not found the strategy to solve it.

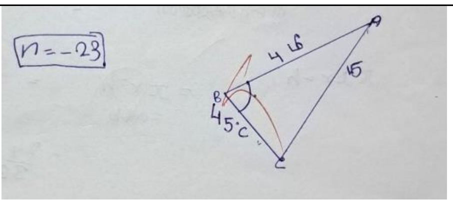
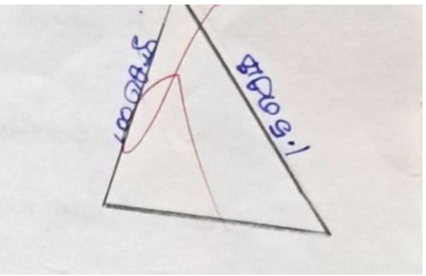
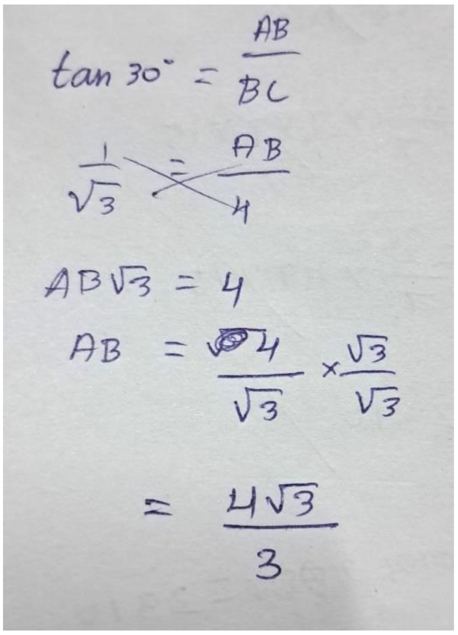
FIGURE 4.6 STUDENTS RESPONSES OF QUESTION 4

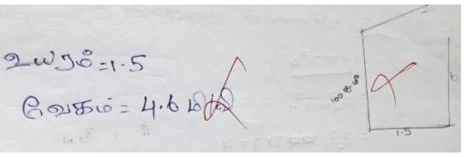
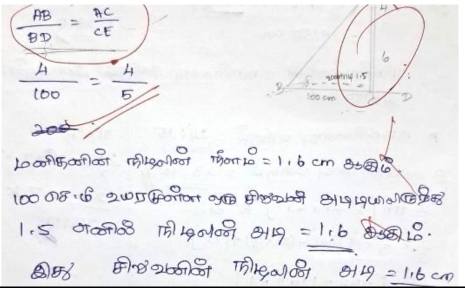


Student's ability in solving HOTS mathematics problems with high and low learning achievement in item 4 is as follows:

<table>
<tr><th>Achievement</th><th>High learning achievement</th><th>Low learning achievement</th></tr>
<tr>
<td>The answers of Students with learning achievement</td>
<td>



Partial responses given by the student</td>
<td>



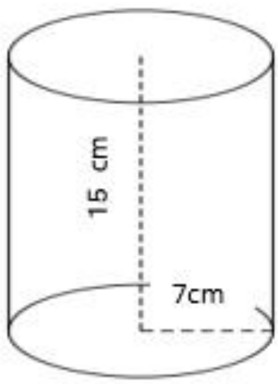

</td>
</tr>
</table>

Reasons for their correct/wrong responses from the interview	• None of the students did not solve this question completely.	• Most of the students could not recognize and understand the given problem. They found it difficult to solve the given problem based on the application of the criterion of similarity.

Question 5:

Containers A and B are shown in the figure.

A



B

a) Which container has more capacity?

b) Find the ratio of capacities of containers A and B.

Results of Question 5:

It is a Decision-making question which is based on Blooms Revised Taxonomy. The researcher expected students to understand, **choose an**

appropriate formula and use it to solve the problem find which container has more capacity and also find the ratio between them.

The table shows how many students got the scores from 0 to 5 in percentage.

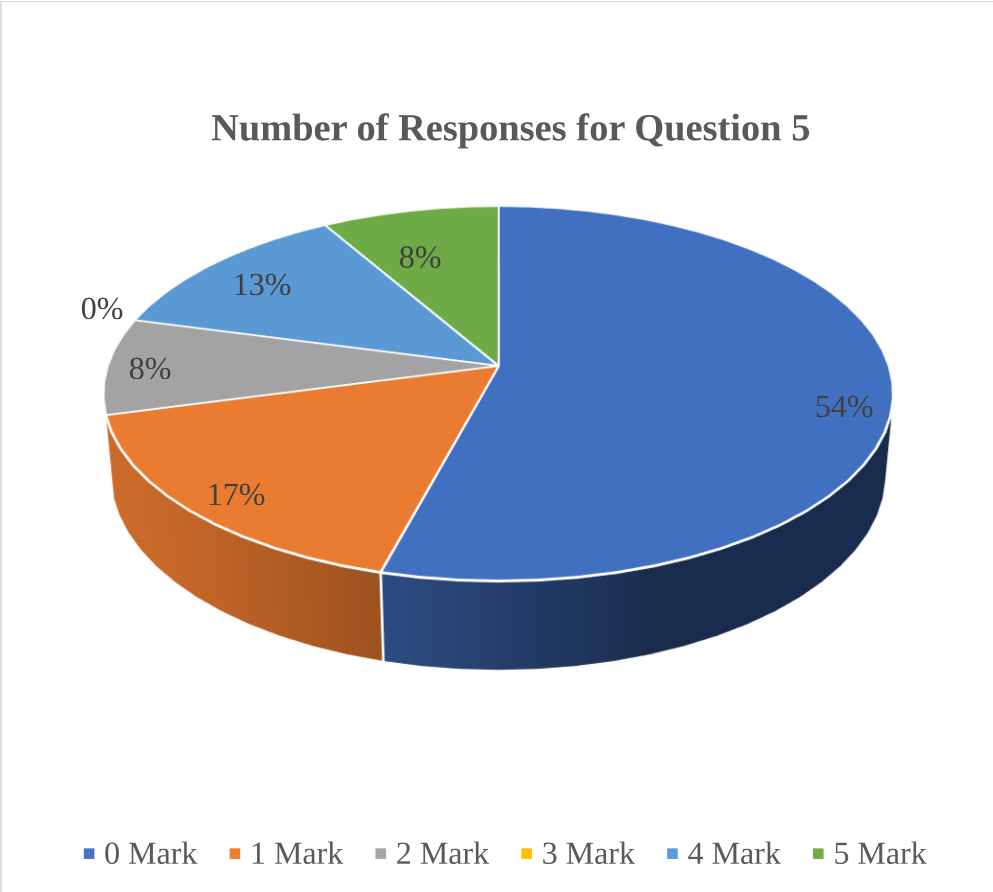
TABLE 4.7 STUDENT'S SCORE FOR QUESTION 5

Marks	**0 Mark**	**1 Mark**	**2 Mark**	**3 Mark**	**4 Mark**	**5 Mark**
Number of Responses (%)	54	17	8	0	13	8

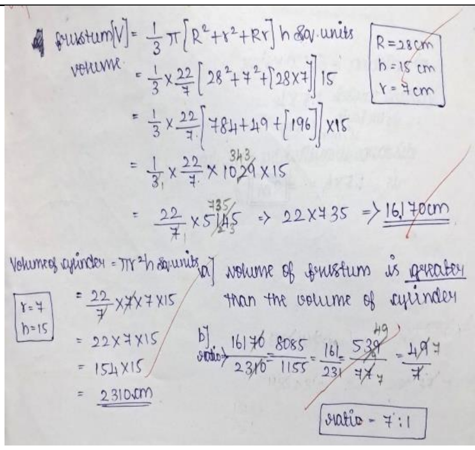
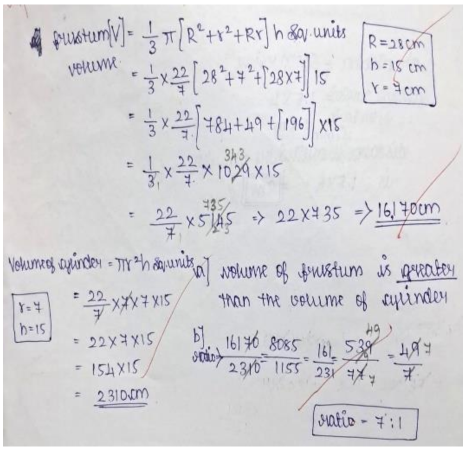
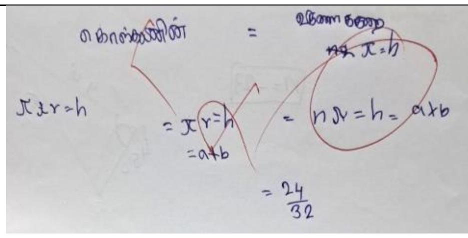
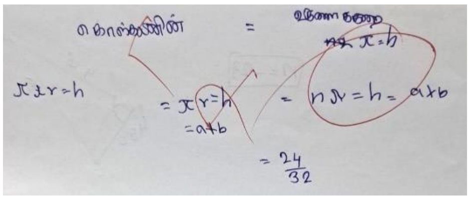
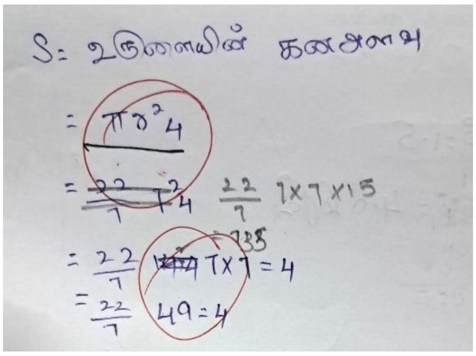
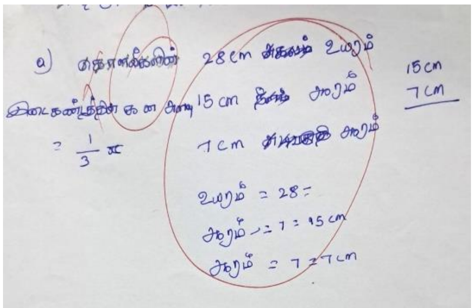
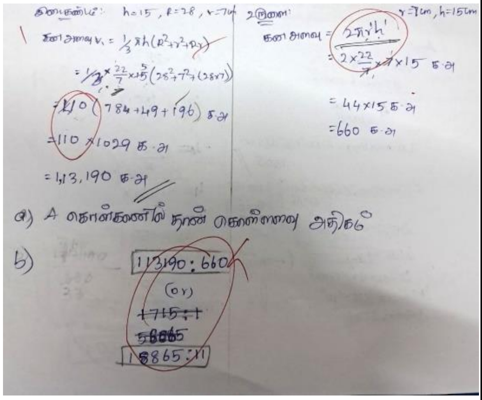
From the above table 4.7, we found that the following findings:

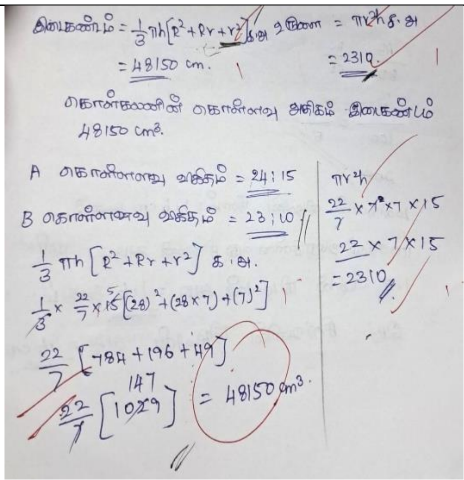
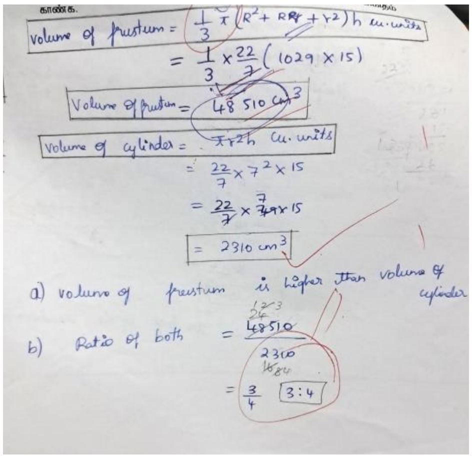
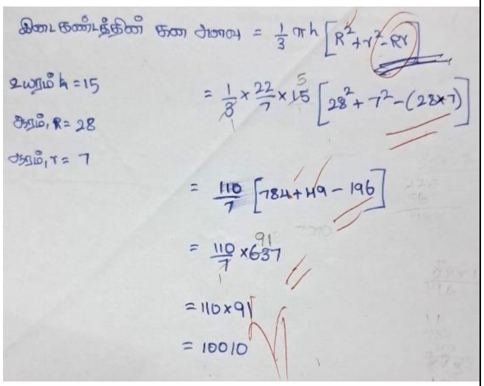
- 54% of students scored 0, 17% of students scored 1, 8% of students scored 2% so in total 79% of students were solved partially to question 5.
- 54% of students did not get any marks for question 5. It reveals that more than half of the students do not read and understand the given problem.
- 71% of students scored 1 or less than 1. They are unable to understand the problem properly and unable to find the correct formulae and solve it.
- 8% of students did not get any marks for question 5. It reveals that more than half of the students do not read and understand the problem.
- 13% of students were unable to solve question 5 completely.

FIGURE 4.7 STUDENTS RESPONSES OF QUESTION 5



Student's ability in solving HOTS mathematics problems with high and low learning achievement in item 5 is as follows:

Achievement	High learning achievement	Low learning achievement
The answers of Students with learning achievement	  Complete responses given by the student	     Students could not recognize the appropriate volume

	  Calculative Errors made by the students	formula for cylinder and frustum  Calculative Errors made by the students
Reasons for their correct/wrong responses from the interview	• 13% of students were unable to solve question 5 completely. • Some students made calculative errors even though they wrote the formulae correctly.	• 54% of students did not get any marks for question 5. It reveals that more than half of the students do not read and understand the given problem.

4.05 RESEARCH QUESTION 4: IS THERE ANY CORRELATION BETWEEN STUDENTS' ABILITY TO SOLVE HIGHER ORDER THINKING MATHEMATICS PROBLEM AND THEIR LEARNING ACHIEVEMENT?

The correlation between students' Ability to solve Higher Order Thinking Mathematics Problems and Learning Achievement was calculated and the same are given in the table.

TABLE 4.8 Correlation of Students' Ability to Solve Higher Order Thinking Mathematics Problems and Learning Achievement

Variables	r – value (Pearson Correlation)	Level of Significance
Student's Ability in Solving Higher Order Thinking Mathematics Problems and Learning Achievement	0.767	Correlation is significant at the 0.01 level

From the Table 4.8, it revealed the value of r is 0.767 which is significant at the 0.01 level and there exists a good positive relationship between students' Ability to solve Higher Order Thinking Mathematics Problems and their Learning Achievement.

4.06 CONCLUSION

This chapter has extensively dealt with the tabulation of calculations about different statistical analyses and the interpretation of results obtained. The findings and conclusions of the study from the analysis have been listed in the next chapter in detail.

CHAPTER V

SUMMARY, FINDINGS, RECOMMENDATIONS SUGGESTIONS FOR FURTHER STUDY AND CONCLUSION

5.01 INTRODUCTION

Mathematics problems are useful for training students to reflect and analyze mathematics. Therefore, students must be taught how to solve problems with appropriate problem-solving.

Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems. Such reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as “Before we go on, are we sure we understand this?” “What are our options?” “Do we have a plan?” “Are we making progress or should we reconsider what we are doing?” “Why do we think this is true?” Such questions help students get in the habit of checking their understanding as they go along.

This chapter summarizes the work done discusses the meaning of the results, addresses the consequences of the results by relating them to the more general conceptual framework of the research topic, suggestions and recommendations for the applications and importance of the research results.

5.02 STATEMENT OF THE PROBLEM

The present study is termed "**STUDENTS' ABILITY IN SOLVING HIGHER ORDER THINKING MATHEMATICS PROBLEM BASED ON LEARNING ACHIEVEMENT – A STUDY**".

5.03 NEED AND SIGNIFICANCE OF THE STUDY

Educational systems worldwide underscore the importance of developing higher-order thinking skills (HOTS) to prepare students for the new challenges of the 21^{st} century. Some pressing issues faced by educators include the ambiguity of the construct; the implementation of HOTS in classroom practices; and the implications for teaching students from linguistically and culturally diverse backgrounds.

We observed that in the National Achievement Survey 2021, The state average of Tamil Nadu in 10^{th} standard in mathematics was 56, which is less than National average (57). we also find that there are 12 learning outcomes taken for assessment and find the highest average and lowest average in mathematics are 34 and 21 respectively.

We have observed that the performance of 10th-standard students in mathematics is not adequate. If the word problem is given then students can understand the given problem, translate the given problem in their own words, select appropriate strategies, to solve by using strategies and also review the solution. Especially, if you make any changes in the textbook problem then, students in high achievers also have been found difficulty in solving the given problems. This study mainly focused on low and high achiever's how they differ in ability to solve higher order thinking problems. From these necessities, we need to find a deeper understanding of how they faced difficulties in solving problems, especially HOT sums.

HOTS's assessment to measure students' abilities in the field of Mathematics and Science internationally has been carried out by Trends in the International Mathematics and Science Study (TIMSS) and the Program of International Student Assessment (PISA). Based on the results of the PISA study in 2009, India was ranked 72 out of 73 participating countries. Therefore, to improve the quality of Indian education, especially in the field of mathematics which refers to international education, it can be done by training students' skills in solving HOTS mathematical problems.

5.04 OPERATIONAL DEFINITION

5.04.01 MATHEMATICS PROBLEM:

A mathematics problem is a question or situation that requires mathematical thinking and problem-solving skills to find a solution. These problems can vary widely in complexity, from simple arithmetic calculations to intricate puzzles that involve advanced mathematical concepts.

5.04.02 PROBLEM SOLVING:

Problem-solving is an essential part of mathematics. Problem-solving is one way to allow students to develop a deeper understanding of mathematical concepts and various ways to represent mathematical solutions. Problem-solving refers to the process of finding solutions to difficult or complex issues or challenges. It involves identifying a problem, understanding its root causes, brainstorming potential solutions, evaluating those solutions, and implementing the best one.

5.04.03 HIGHER ORDER THINKING SKILLS (HOTS)

Higher order thinking is not a new concept and it concerns synthesis, evaluation, interpretation, hypothesising, prediction, conjecture, critical thinking and judgement. It is complex and involves reflection, self-regulation, testing of ideas, and problem-solving. In HOTS mathematical problem is a non-routine mathematical problem that contains elements of analysis, evaluation, and creation.

5.04.04 LEARNING ACHIEVEMENT

Learning Achievement or Academic achievement is the outcome of education, the extent to which a student has achieved their educational goals. Learning Achievement is commonly measured by examinations or continuous assessment.

The half-yearly exam scores of 10th students in Mathematics subjects have been taken into account for the measurement of the academic performance of the student.

5.05 OBJECTIVES OF THE STUDY

- To assess the ability of students to solve Higher Order Thinking (HOTS) mathematics problems with high and low learning achievement based on Krulik and Rudnick's problem-solving.
- To find the relationship between the ability of students to solve Higher Order Thinking (HOTS) mathematics problems and learning achievement.

5.06 RESEARCH QUESTIONS

1. What are the Abilities of Students to Solve Higher Order Thinking Mathematics Problems on Learning Achievement?
2. What are the abilities of students to solve Higher Order Thinking Skills (HOTS) mathematics problems among high and low learning achievement based on Krulik and Rudnick's problem-solving?
3. Why do students lack in solving HOT Mathematics Problems?
4. Is there any correlation between students' Ability to solve Higher Order Thinking Mathematics Problems and their Learning Achievement?

5.07 TOOL USED IN THE STUDY

This research includes test questions and interview items. we conducted a workshop for preparing the test items and also the interview schedule of the 10th standard. The test questions in this study consist of 5 Higher Order Thinking Skill-based word problems. These test questions were prepared with the elements of Revised Blooms Taxonomy questions like analysis, evaluation, and creation and also which were not routine mathematical problems. We prepared an interview schedule based on Krulik and Rudnick's problem-solving steps.

5.08 SAMPLE

The population of the present study was tenth-standard students in Ranipet District. In this study, the researcher used the Purposive - Stratified Sampling Technique to select the schools. Also, the researcher has chosen 3 schools which best perform in the 10th public exam in the Ranipet District.

5.09 MAJOR FINDINGS OF THE STUDY

The major findings of the present study were listed as follows:

- The average score of Learning Achievement in the half-yearly exam and Average Performance in the solving-based test are 60.96 and 18.67 respectively.
- The average achievement of the Low and High Achievers in the half yearly exam are 34.08 and 87.83 respectively. Also, we found that the test result of HOT HOT-based test, the Average performance of the Low and High Achievers are 2.33 and 35 respectively.
- In question 1, 42% of students got 0 marks, 6% of students got 1 mark, 4% of students got 2 marks, 4% of students got 4 marks and only 25% of students got full marks. Many students didn't know "where question 1 comes from" and they didn't understand the concept in which the topic or sub-topic was covered. 42% of students did not know what to do and some of them wrote irrelevant answers.
- All students are unable to solve question 2 completely. Many of them do not recognise that "the given question in which topic or subtopic is covered?". Also, they have not understood the question and may not find the relevance of the concepts. All students are unable to solve question 2 completely. 92% of students do not recognise that "the given question in which topic or subtopic is covered?". Also, they have not understood the question and may not find the relevance of the concepts. Even high achievers did not choose the formulae/steps for solving. Low achievers did not respond anything for this question.
- 58% of students got 0 marks and 17% of students got 1 mark, so 75% of students were unable to understand, plan and solve question 3. 83% of Students answered irrelevantly or selected wrong

formulae. They were unable to solve due to not understanding the given calculation properly, not knowing how to convert it into a mathematical statement, not knowing how to choose the optimal formula, and not using the formula properly.

- 79% of students got 0 marks, 8% of students got 1 mark, and 13% of students got 2 marks so 100% of students did not solve question 4 completely. It reveals that most of the students could not recognize and understand the given problem. Most of the students could not recognize and understand the given problem. They found it difficult to solve the given problem based on the application of the criterion of similarity.
- 54% of students scored 0, 17% of students scored 1, 8% of students scored 2% so in total 79% of students were solved partially to question 5. 54% of students did not get any marks for question 5. It reveals that more than half of the students do not read and understand the given problem.
- It revealed the value of r is 0.767 which is significant at the 0.01 level and there exists a good positive relationship between students' Ability to solve Higher Order Thinking Skills (Hots) Mathematics and their Learning Achievement.

5.10 INTERPRETATION AND DISCUSSION

The researcher conducted interviews related to the answers of students based on their learning achievements. Interview activities are conducted to determine student's ability to solve Higher Order Thinking Skills (HOTS) mathematics problems. Analysis of student's ability to solve HOTS mathematics problems based on Krulik and Rudnick's problem-solving steps as follows:

Low-achieving students:

This study reveals that the overall performance in solving HOT-based Mathematics problems was not good and not appreciated level. Here are the reasons for the lack of solving HOT mathematics problems as follows:

1. They were not capable of reading the given problem because their language proficiency was not adequate.
2. They were not able to recognize 'which topic or subtopic is covered?' because either they were unable to find the key terms in the word problem or they did have not enough practice in such topics.
3. They were not able to find the relevant concepts because lack of understanding of the question.
4. They were not able to recall the formula to solve the problem because they did have not enough practice or mnemonic techniques for recalling formulae.
5. Some students find the correct formulae, but they could not substitute properly or they could not continue steps one by one because they did not find given values to the corresponding variables or they were lack of solving skills.

High-achieving students:

1. Even high achievers did not choose the correct formulae for the given problem or they did not continue the steps to solve it because they did not have enough exposure to solve lengthy word problems in Sum of arithmetic progression, Sum of squares of n natural numbers and Congruency.

2. Most of the students could not recognize and understand the given problem. They found it difficult to solve the given problem based on the application of the criterion of similarity.

Students experience obstacles in determining the find the answer and also cannot understand the concepts of Function, Equation, Arithmetic Progression, Squares of natural numbers, Congruency and Volume of 3D. This makes it difficult to determine the next step in solving. Students are not able to continue their work, so students with low learning achievement cannot solve HOTS math problems. It also reveals that learning achievers both could not perform well in overall. The teachers believe that students with high learning achievement were able to solve HOTS math problems but it did not happen in real.

5.11 CONCLUSION

Based on data analysis that has been done by researchers about the ability of students to solve Higher Order Thinking Skills (HOTS) mathematics problems with high and low learning achievement, it reveals that high learning achievement students have better HOTS-solving ability than low learning achievement students but the overall performance is not sufficient level.

High learning achievement students can do the problem-solving process correctly and with the right answer, but low learning achievement students are unable to recognize the problem, unable to find the strategy, select the wrong steps and cannot continue the next step, so the low learning achievement students cannot find the right answer.

Most of the high learning achievement students where find the difficult to solve problems which are in Sum of an Arithmetic Progression, sum of squares of first n natural numbers and congruency triangle.

5.12 EDUCATIONAL IMPLICATION

Today the world is moving towards achieving 21st-century skills. In this problem solving is one of the important skills in 21st century skills. The purpose of this study was to describe student's ability to solve HOTS mathematics problems with high and low learning achievement. This study reveals that both learning achievers differ in solving HOT, but overall performance is not appropriate level.

This study will help the teachers to understand better with high learning achievement students could not perform well, because when we make some changes to textbook problems, students are unable to understand, unable to plan for the right strategy and solve the problem.

This study reveals that students with low learning achievement did not understand what was given in the problem, what they wanted to find, whether the word problem was clearly defined or not and also, they were unable to find where it came from. In this, the STAR strategy (Search - Translate-Answer- Review) or Krulik and Rudnick's steps ((Read and think - Explore and plan - Select a strategy - Find an answer - Reflect and extend)) to solve the word problems can be used by the teachers in their teaching-learning process.

Now we are in a crucial stage to build the student's capacity like solving the HOT-based problem with different contexts and posing different questions. In that case, teachers ask different kinds of problems in the classroom and give enough space to solve the non-routine problems and pose questions. Hence School Education Department and SCERT will take taken necessary steps for students to solve HOT sums in their classes frequently.

Teachers ensure frequently in solving HOT problems capacity of students in their classes. Teachers may also assess the capacity of students not only to solve textbook problems than beyond textbook problems. Teachers can have a list of difficult topics for further learning improvement or further teaching.

5.13 SUGGESTIONS FOR FURTHER RESEARCH

The researcher has pointed out the following recommendations after consideration of this research:

1. 10th-standard students were chosen by the researcher as the sample of this study. Similar research can be done for the students in class 8 and class 12.
2. The research tool of this study has been focused on assessing the concepts were Relation and Functions, the Sum of an Arithmetic Progression, the Sum of Squares of first and Natural Numbers and the Congruency of triangles and the Volume of 3-dimensional figures. Similar research can be done for other topics or units in the 10th standard.
3. The research can be done to assess a better understanding of teachers in Bloom's taxonomy and Higher Order Thinking problems.
4. The School Education Department and SCERT are to be taken necessary steps to improve the teacher's understanding of HOT problems.